

DETERMINATION OF LORENZ EQUATION FOR MEASURING INCOME INEQUALITY FOR MIZORAM STATE, INDIA

Dr. JV. Nunchunga¹

¹Associate Professor of Economics, Govt. J. Thankima College, Aizawl, Mizoram

ABSTRACT

The paper explained the widely employed Lorenz curve for measuring income inequality in general and thus used for measuring income inequality for Mizoram in particular. The paper examined six models/equations for fitting the income share of deciles using SPSS software, and considered the Cubic model as the best fit for the Mizoram data)

Keywords: Lorenz curve, BPL, Bowley's allocation of sample, optimum sample size, income per capita, and Deciles

1. Introduction:

Mizoram is one of the states of India, sandwiched between $92^{\circ} 15' E$ to $93^{\circ} 29' E$ longitudes and $21^{\circ} 58' N$ to $24^{\circ} 35' N$ latitudes. It is bounded by Myanmar in the east and south, Bangladesh and Tripura state in the west, Assam state and Manipur state in the north. It has an international boarder of 404 km and 318 km with Myanmar and Bangladesh respectively. The length of its inter-state boarder with Assam, Tripura and Manipur are 123 km, 66 km and 95 km respectively. The state has a geographical area of 21,081 Sq.km with a population of 10,91,014 in 2011.

2. Lorenz curve:

The Lorenz curve, widely used to represent and analyse the size distribution of income and wealth, is defined as the relationship between the cumulative proportion of income units $F(x)$ and the cumulative proportion of income received $F_1(x)$, when units are arranged in ascending order of their magnitude. Lorenz proposed this curve in 1905 in order to compare and analyse inequalities of wealth in a country during different epochs, or in different countries during the same epoch.

¹ Associate Professor of Economics, Govt. J. Thankima College, Aizawl, Mizoram

An income unit x is a random variable with the probability density function $f(x)$ given by²

$$F(x) = \int_0^x f(x) dx \quad - \quad (1)$$

Further, it was assumed that

$$\frac{dF(x)}{dx} = f(x) \quad - \quad (2)$$

where $F(x)$ can be interpreted as the proportion of units having an income less than or equal to x . obviously $0 \leq F(x) \leq 1$. Further, if it was assumed that the mean μ of the distribution exist, the first moment of the distribution function of x is defined as

$$F_1(x) = \frac{1}{\mu} \int_0^x x f(x) dx \quad - \quad (3)$$

Where $0 \leq F_1(x) \leq 1$ and $F_1(x)$ is interpreted as the proportional share of total income of the unit having an income less than or equal to x . If $f(x)$ is continuous, the derivative of $F_1(x)$ exists and is given by

$$\frac{dF_1(x)}{dx} = \frac{x f(x)}{\mu} \quad - \quad (4)$$

which imply that $F_1(x)$ is a monotonically non-decreasing function³ of x .

The Lorenz curve is the relationship between the variable $F_1(x)$ and $F(x)$. The curve can be plotted by generating the values of $F(x)$ from equation 1 and 3 by assigning arbitrary values to x . The curve is represented in a unit square figure. The ordinate and abscissa are being $F_1(x)$ and $F(x)$ respectively.

By means of equation 2 and 3 we obtain the slope of the Lorenz curve as

² Income can be negative for some unit but it is assumed to be non-negative for the convenience of analysis.

³ If successive increase in x always lead to successive increasing in $F(x)$, that is, if $x_1 > x_2 \Rightarrow f_1(x_1) > f_2(x_2)$, the function is said to be strictly monotonically increasing function, but some writers prefer to define an ascending step function, not an increasing function, but a *monotonically non-decreasing function*.

$\frac{F_1(x)}{F(x)} = \frac{x}{\mu}$, which is always positive for positive income. Similarly, the second derivative of the curve is

$$\frac{d^2 F_1(x)}{F(x)^2} = \frac{d}{dF(x)} \left(\frac{dF_1(x)}{dF(x)} \right) = \frac{1}{\mu f(x)} \geq 0, \quad - (5)$$

which is also positive. These two derivatives imply that the slope of the curve is positive and increases monotonically, that is, the curve is convex to the $F(x)$ axis, and from this it follows that $F_1(x) \leq F(x)$. When $F_1(x) = F(x)$, the condition is known as egalitarian condition.

3. Objective:

The paper tried to identify the most suitable functional form for the percentile distribution of income for the state of Mizoram.

4. Sample size determination:

For determining the sample size, the percentage of people Below Poverty Line (BPL) is used. The percentage of BPL lies between the 15% and 19.49%. This will be used for determining the sample size. The degree of precision is usually determined in terms of-

- (i) The margin of error permissible in the estimate (d)
- (ii) The confidence coefficient $(1-\alpha)$ with which we want this estimate to lie within the permissible margin of error.

In this study it will be sufficient if the level of d is 5% and $(1-\alpha)$ is 95%.

Here the population is divided into two mutually exclusive groups – Below poverty line (BPL) and Above poverty line (APL). From Probability theory, we know that

$$P(|p-P| \geq d) = \alpha,$$

Simple random sampling is assumed, and p is taken as normally distributed. We have

$$\sigma_p = \sqrt{\frac{N-n}{N-1}} \sqrt{\frac{PQ}{n}}. \text{ Hence the formula that connects 'n' with the desired}$$

degree of precision is $d = t \sqrt{\frac{N-1}{N-1}} \sqrt{\frac{PQ}{n}}$, where t is the abscissa of the normal curve that cuts off an area of α at the tails. Solving for 'n', we get

$$n = \frac{\frac{t^2 PQ}{d^2}}{1 + \frac{1}{N} \left(\frac{t^2 PQ}{d^2} - 1 \right)},$$

For practical use, an advance estimate p of P is substituted in this formula. If N is large, a first approximation is $n_0 = \frac{t^2 pq}{d^2}$, If $\frac{n_0}{N}$ is negligible, n_0 is the satisfactory approximation of n . If not, the sample size is

$$n = \frac{n_0}{1 + \frac{n_0 - 1}{N}}$$

Or
$$n = \frac{n_0}{1 + \frac{n_0}{N}} .$$

In Mizoram there are 1,71,631 families, the required sample size is

$$n_0 = \frac{2^2 (.1949)(.8051)}{(.05)^2} = 252 .$$

5. Methodology of sample collection:

In Mizoram, there are 8 districts and 22 Rural Development Blocks. For the purpose of data collection, it was given a careful thought whether districts will be taken as strata or not. It was decided that while selecting sample district may be considered as strata.

For allocation of sample, Bowley's allocation of sample is used. Before collecting sample a sampling frame consisting of villages is prepared and some villages are selected using cluster technique of sample survey. After a village is selected sampling units are listed out and the final samples are selected using simple random sampling with small application of judgment sampling technique.

The samples were collected in the year 2006 covering 1579 persons. The optimum sample size for each district and the sample size collected are given in the appendix 1.

6. Estimation of monthly income per capita:

The data given in this literature is the monthly income per capita. For estimating household income an economy is divided into 3 sectors viz; primary, secondary and tertiary sector. The production of Primary sector is calculated by production approach except income from manual daily labourers at 2006 current year prices.

For estimating the contribution of secondary sector, a mixture of income and expenditure is used. For mining & quarrying income method is used with a deduction of input costs of 33% from the total income. For construction expenditure method is used and for others income method is employed. For Tertiary sector, income method is used to arrive at the final figure.

7. The distribution of income:

The distribution of income giving the percentage and decile share of the sampled households is given in the following table.

Table No.1: (Distribution of income in Mizoram)

Deciles	Deciles' SHARE %	Cumulative % share
First 10%	0.82	0.82
Second 10%	1.76	2.58
Third 10%	2.38	4.96
Fourth 10%	3.58	8.54
Fifth 10%	5.03	13.57
Sixth 10%	7.11	20.68
Seventh 10%	9.09	29.77
Eighth 10%	12.61	42.38
Ninth 10%	16.67	59.05
Top 10 %	41.22	100.27

Source: JV. Nunchunga (2007)

8. Curve Determination:

Six curves are plotted using the SPSS software. All of them are statistically significant and the model summary and parameter estimates are given in table 2.

Table 2: (Model summary and parameter estimates)

Equation	Model Summary					Parameter Estimates			
	R Square	F	df ₁	df ₂	Sig.	Constant	b ₁	b ₂	b ₃
Cubic	0.991	229.289	3	6	.000	-9.688	1.115	-.026	.000
Power	0.982	425.105	1	8	.000	.006	2.020		
Quadratic	0.968	106.612	2	7	.000	12.532	-.856	.016	
Compound	0.966	226.576	1	8	.000	.930	1.050		
Growth	0.966	226.576	1	8	.000	-.073	.049		
Exponential	0.966	226.576	1	8	.000	.930	.049		

Generated by SPSS

9. Conclusion:

From table 2, it is appeared that Cubic model is the best fit for the data. The percentage share by each group of income y and the cumulative percentage of persons receiving income x is linked by cubic model of the form

$$Y = - 5.3015 + 0.7468x - 0.18x^2 + 0.0002x^3$$

with R – square being 99.1%. All the coefficients are statistically significant.

APPENDIX I

(Showing the optimum sample, Actual sample size and No. of persons covered)

Districts	Aizawl	Lunglei	Champhai	Kolasib	Lawngtlai	Mamit	Saiha	Serchhip	Mizoram
Optimum sample	98	37	30	16	21	18	17	15	252
Actual Sample	98	36	27	19	26	16	15	19	256
No. of persons covered	586	218	186	109	191	82	97	110	1579

Source: JV. Nunchunga (2007)

REFERENCES

- Anand, Sudhir (1983) '*Inequality and Poverty in Malaysia: Measurement and Decomposition*', New York: Oxford University Press
- Atkinson A.B.(1975) '*The Economics of Inequality*' Oxford University Press,
- Chaubey, P.K.(1996) '*Economic Inequality: Theory of Measurement*', New Delhi: Indian Economic Association Trust for Research and Development,
- Directorate of Economics & Statistics (2003), '*Village Level statistics of Mizoram-2003*', Aizawl
- Directorates of Economics & Statistics (2004), '*State domestic product of Mizoram*', Aizawl
- Kumar, Ashutosh (2003) '*Economic development and income distribution*' , New Delhi : Deep and Deep Publications
- Nunchunga, JV '*A mathematical study on the income distribution and estimation of inequality*'. (Unpublished Ph. D thesis), 2007, Mizoram University
- Prakash, BA (2004) '*Kerala's Economic Development*' New Delhi: Sage Publication, Second edition.
- Sen, Amartya (1973a), '*on Economic Inequality*', Delhi: Oxford University Press.
- Subramaniam, S. (ed.) (1997), '*Introduction: The Measurement of Inequality and Poverty*', Oxford University Press.