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MODELLING A REVENUE MAXIMISING RELATIONSHIP BETWEEN THE VOLUME OF A TRADEABLE ASSET SOLD AND THE APPRECIATION IN THE ASSET'S PRICE

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ABSTRACT

Through the application of financial mathematics, this paper explores how mathematical modelling can be used to benefit from favourable conditions in financial markets like asset appreciations. Given that assets should be sold upon the appreciation of their price, this paper aims to find a mathematical relationship between the increase in the price of an asset and the specific amount that should be liquidated in order to maximise revenue. After analysing the data, this paper concludes that if Δpdenotes the appreciation in price, $L(\Delta p)$ the amount liquidated, T is a constant representing the maximum possible value of Δp and m is the minimum value of $L(\Delta p)$, then

$$L(\Delta p) = \frac{(100-m)(n+1)\Delta p^{(n+1)}}{T^{(n+1)}(n+1)} + m \textit{ such that}$$

$$n \textit{ (constant acting as an exponent) is greater than } u \textit{ where } u \textit{ is the minimum value of } n \textit{ at which } L'(\Delta p) = \frac{-(ln(m)-ln(100))\times 100}{T}$$

The results from this modelling process are highly applicable in day trading, where prices of assets such as foreign exchange and cryptocurrency fluctuate in the short run, and in conceptualising trading strategies which favour short-term revenue inflows.

Keywords: Revenue Maximisation, Financial Markets, Asset Liquidation, Modelling, Price Appreciations

Introduction

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Is there an ideal relationship between the amount of a tradeable asset that should be liquidated and the change in the asset's price? If so, what does this relationship look like, how does it vary with fluctuations in price and is it possible to maximise the revenue generated via the proportion of asset sold at different prices? To investigate these questions, this paper aims to apply linear, quadratic, sinusoidal, logistic, exponential and power mathematical models to explore different relationships between changes in price and proportion of asset that should be liquidated. The results from this investigation can be applied in trading strategies wherein traders can capitalise on short-term appreciations to ensure that every marginal trade generates revenue. It is found that a power model yields the most effective relationship.

Methodology:

To find a potential revenue maximising relationship between the change in price and the amount of an asset that should be liquidated, I first defined the respective thresholds to which the price can change through the application of past-price statistics.These are essentially parameters vis-à-vis the potential proportions liquidated as per potential changes in price. After establishing fixed parameters vis-à-vis the proportions liquidated and potential changes in price and considering different types of increasing functions, I derived functions that determine a relationship between the change in price and the amount of an asset that should be liquidated. For mathematically rigorous calculations, the Casio CG-50 Graphics Display Calculator was used along with Wolfram Alpha for more challenging derivatives and equations. For the purpose of this paper, I considered linear, quadratic, sinusoidal, logistic, and exponential functions. Furthermore, I derived the equation that describes how selling different quantities of an asset impacts revenue models. To find the ideal model for liquidation, I analysed the relationship between the derivatives of each model and revenue through the use of basic differential calculus. Lastly, adapting data from Yahoo Finance,[1]I utilised the prices of BITCOIN from 22nd December 2021 to 23rd March 2022 to test the models. In this testing process, future price values were simulated using Microsoft Excel's built-in random integer generator.

Conceptual Overview:

[1]Yahoo! Finance. "Bitcoin USD (BTC-USD) Price History & Historical Data - Yahoo Finance." *Finance.yahoo.com*, 23 Mar. 2022, finance.yahoo.com/quote/BTC-USD/history?period1=1640131200&period2=1647993600&interval=1d&filter=history&frequency=1d&includeAdjustedClose=true. Accessed 2 Apr. 2022.

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The fundamental principal behind all financial transactions involving tradeable securities is to sell an asset at a higher price than its purchase price. The amount an asset appreciates by can be expressed as

$$\Delta p = p - p_0 \tag{1}$$

where p_0 represents the initial purchase price and p represents the price at which the asset is traded in a particular trade at a particular point in time. In this equation, if $p > p_0$,then there is an increase in price (the scope of discussion is limited to appreciation). For every trade made where $p > p_0$, the trader earns a revenue, R, equal to

$$R = Vp \tag{2}$$

where V is the volume of an asset purchased.It is assumed that the trader retains all revenue generated per trade without paying any share to a third party. However, in (2), 100% of V is being liquidated at p_t (where t denotes time). If the price of the asset appreciates to $p_{(t+1)}$ which is greater than p_t, the 100% liquidation detailed in (2) leads to lost potential profits. In order to maximize potential profit, the volume of the asset that would be liquidated should be related to the extent of the appreciation. The relationship can be expressed as a mathematical function $L(\Delta p)$ where L represents the proportion of V liquidated with respect to the extent to which the price appreciates or the magnitude of Δp.

Conditions for Modelling:

The assumptions behind the modelling of $L(\Delta p)$ are as follows. Firstly, Δp is always greater than 0, ergo liquidation only occurs when the price of the asset appreciates. In addition, Δp is considered as the only variable that impacts the proportion of the asset liquidated as all other influences on trading are held constant. As liquidation occurs for every positive value of Δp, as long as the price appreciates, liquidation occurs. $L(\Delta p)$ must always be greater than m, which is the minimum proportion of V that must be liquidated. It is theoretically impossible to liquidate 0 shares and given that liquidation relates to Δp, which is greater than 0, as clarified above, m becomes the L intercept (conventional y-intercept on the cartesian plane). Thus, $m = L(0)$. Lastly, the model relies on a maximum threshold value, T, with respect to Δp. Should $\Delta p = T$, 100% of Vwill be liquidated, and thus$L(T) = 100$. Both m and T are considered to be essential constants for the modelling process.

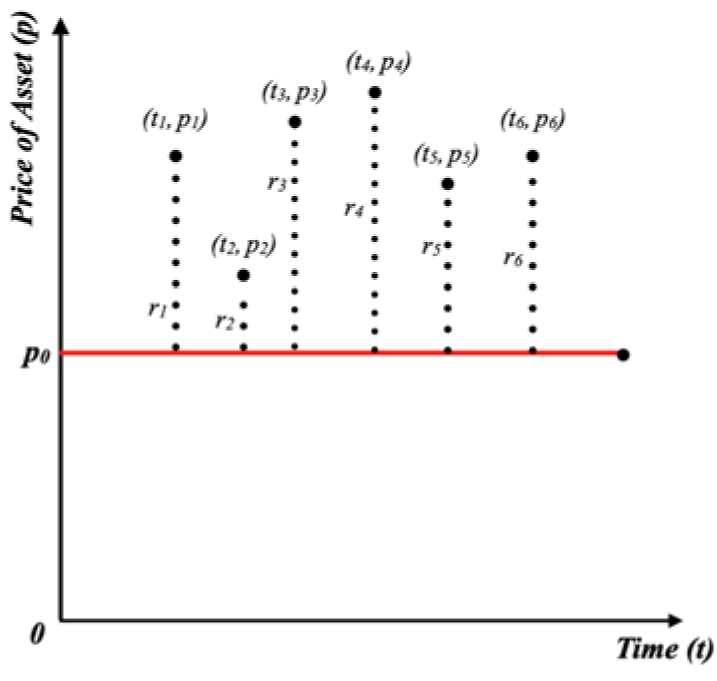
Calculating m and T:

Calculating the value of m is relatively straightforward. Let us assume that a brokerage specifies that M units of an asset must be sold per trade. By converting M as a proportion of V we get

$$m = \frac{100M}{V} \tag{3}$$

On the other hand, the value of T should be determined based on the previous movements of the asset's price since T is a maximum threshold for Δp. The concept of residuals may be applied in this instance. In statistics, the residual is a predicted y value subtracted from the actual y value. In our case, we may consider the residual to be the positive difference between p_0 and past values of $p$per defined periods of time. It is once again assumed all past values of p are greater than p_0. The same is illustrated in Figure 1.

Figure 1: [Upper] Residual Price Values



T is determined by the mean of all the positive residual price points taken per period of time. In order to calculate T, we must subtract p_0 from all previous prices values that are greater than p_0 and calculate the mean of the differences. This is shown below mathematically.

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$$T = \frac{(\sum_{i=1}^{w}(p_i - p_0))}{w} \tag{4}$$

where w is the total number of residual price points taken. This value of T ensures that the point at which 100% of the asset is liquidated is based on previous values of p rather than being abstract and thereby also ensures that price and $\Delta p$are the sole determinants of $L(\Delta p)$. Therefore, the domain of $L(\Delta p)$ is $\{\Delta p | \ 0 < \Delta p \leq T | \Delta p \in R\}$ and the range is $\{L(\Delta p) | \ m < L(\Delta p) \leq 100 | L(\Delta p) \in R\}$.

Linear Model:

The simplest relationship between $L(\Delta p)$ and Δp is a linear relationship. A linear relationship implies that $L(\Delta p)$ and Δp are directly proportional to each other. Given that $L(0) = m$ and the function $y = kx$ represents direct proportionality between $L(\Delta p)$ and Δp, we may modify the linear function $y = ax + b$ (a is the gradient and b is the y-intercept) in the following manner:

$$L(\Delta p) - m = k\Delta p \tag{5}$$

$$L(\Delta p) = k\Delta p + m \tag{6}$$

Here m is the y-intercept and as per the aforementioned domain, $L(T) = 100$ thus,

$$100 = kT - m \tag{7}$$

$$\frac{(100 - m)}{T} = k$$

$$L(\Delta p) = \frac{(100 - m)}{T}\Delta p + m \tag{8}$$

The linear relationship is depicted in Figure 2 for $T = 200$ and $m = 3$.

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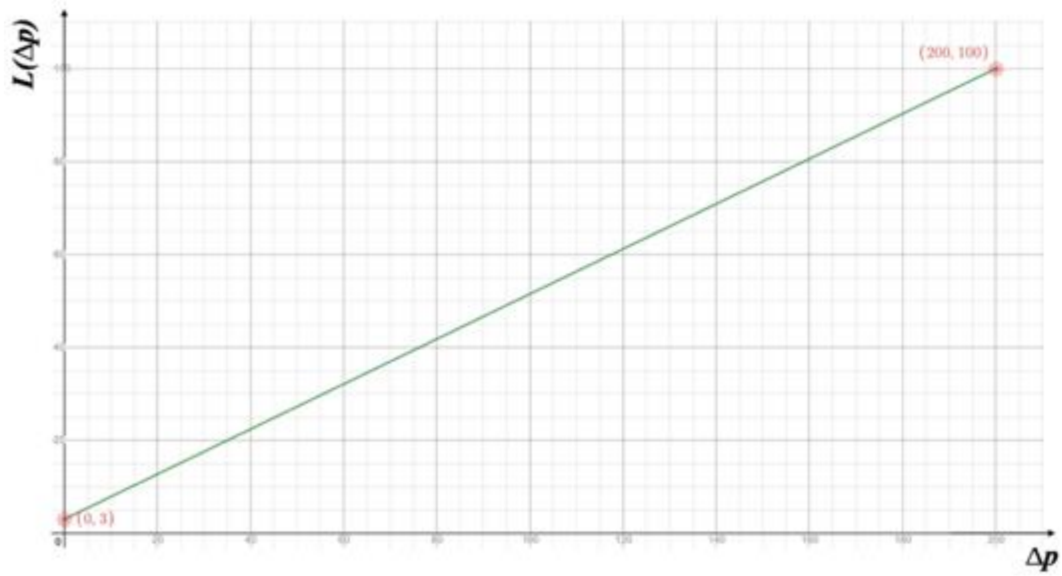
Figure 2: The Linear Model



As evident in Figure 2, the line intersects the y axis at $(0, m)$ or$(0,3)$. The exact function shown is $L(\Delta p) = 0.485\Delta p + 3$. Given the linear nature of the model, the gradient of the line is simply $\frac{(100-3)}{200}$ or 0.485. In this instance, an additional 0.485% is liquidated for every 1 unit increase in Δp.

Quadratic Model:

The traditional quadratic equation can be expressed as a function in the form:

$$f(x) = ax^2 + bx + c \tag{9}$$

The maximum of $L(\Delta p)$ will be at $(T, 100)$ where all units are liquidated.

Given that m is $L(0)$, $c = m$ as both are the y intercepts in the cartesian plane. As the axis of symmetry of any quadratic function is $\frac{-b}{2a}$, it may be used to derive the full quadratic model:

$$T = \frac{-b}{2a} \tag{10}$$

$$b = -2aT$$

Using the substitution,

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$$aT^2 - 2aT(T) + m \ = \ 100 \tag{11}$$

$$a(-T^2) = \ 100 - m$$

$$a \ = \frac{(100 - m)}{-T^2}$$

$$b \ = \ -2 \times \frac{(100 - m)}{-T^2} \times T$$

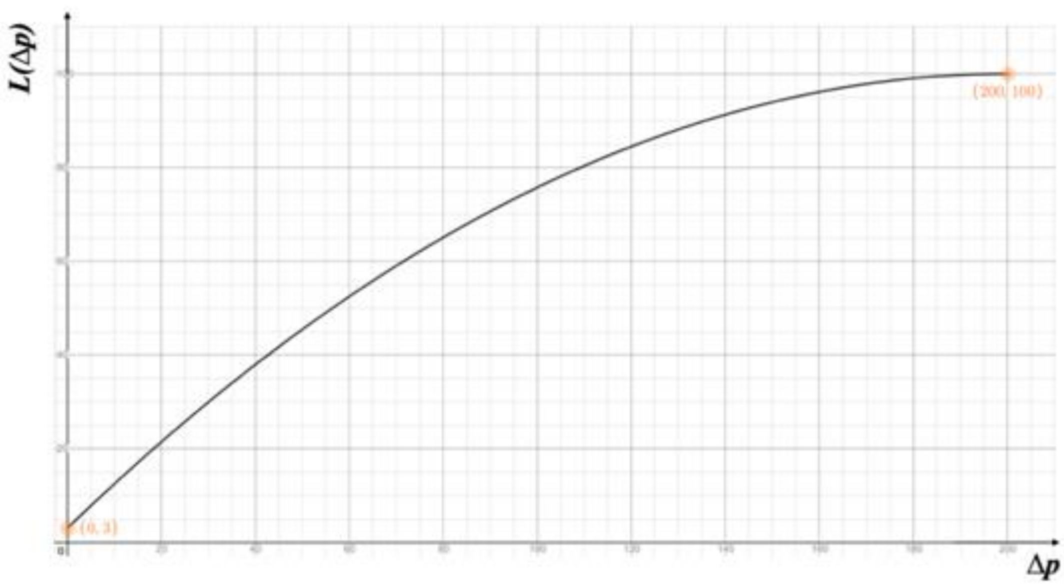
$$b \ = \frac{2(100 - m)}{T}$$

Thus, the complete model is:

$$L(\Delta p) = \frac{(100 - m)}{-T^2} \Delta p^2 + \frac{2(100 - m)}{T} \Delta p + m \tag{12}$$

To elucidate this model, setting m as 3 and T as 200, the graph of $L(\Delta p)$ is shown in Figure 3.

Figure 3: The Quadratic Model



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This function intersects the y axis at $(0, m)$ or $(0,3)$ and has a maximum at $(T, 100)$ or $(200,100)$. Given these particular values of m and T, the exact function shown above is $L(\Delta p) = (2.425 \times 10^{-3})\Delta p^2 + 0.97\Delta p + 3$.

Sinusoidal Model:

A sinusoidal model is characterised by continuous smooth periodic oscillations. The sinusoidal model used in this paper is the Sine Model which follows the general form

$$f(x) = \text{a} \sin\big(b(x-c)\big) + d \tag{13}$$

This formis derived from $y = \sin x$ by a vertical stretch with a scale factor of a, a horizontal stretch with a scale of factor of $\frac{1}{b}$, followed by a horizontal translation of c units and a vertical translation of d units.[2] Since there is periodic oscillation, the maximum of the function occurs at half the period as per the domain at $L(T)$. Thus, only a small segment of the overall function is considered as $L(\Delta p)$, which is constrained by the set domain and range. The function can be derived through the following steps:

$$a = \frac{L(T) - m}{2} = \frac{100 - m}{2} \tag{14}$$

$$d = \frac{L(T) + m}{2} = \frac{100 + m}{2} \tag{15}$$

b is $\frac{2\pi}{Period}$ and since T is half the period, b can be simplified accordingly

$$b = \frac{2\pi}{2T} = \frac{\pi}{T} \tag{16}$$

We know that the maximum of the sine function will be when $\Delta p = T$ and the minimum is when $\Delta p = 0$. The difference between the maximum and minimum is half the period, T. As the distance between the midpoint and the maximum is the horizontal shift from the L axis, the distance can be calculated as$\frac{T}{2}$. Hence, $c + \frac{T}{2} = T$, so

$$c = \frac{T}{2} \tag{17}$$

[2]Haese, Michael, et al. *Mathematics Core Topic HL 1*. 2019. 1st ed., Haese & Harris Publications, 2019.

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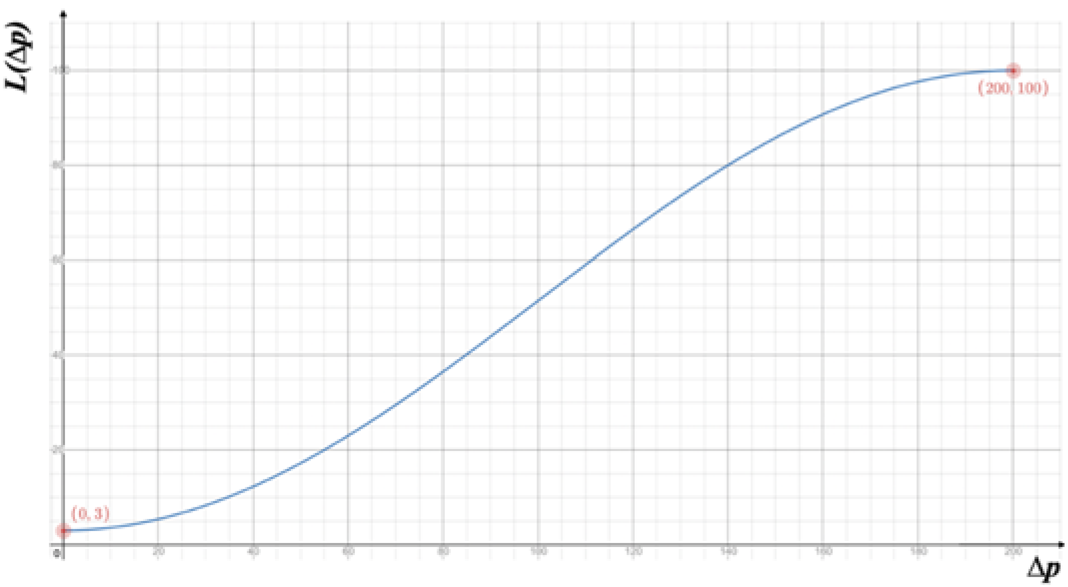
The complete half-wave function per the appropriate domain and range is:

$$L(\Delta p) = \frac{(100-m)}{2}\sin(\frac{\pi}{T}(\Delta p \ - \frac{T}{2})) + \frac{(100+m)}{2} \qquad (18)$$

Substituting T as 200 and m as 3, we obtain Figure 4.

Figure 4: The Sinusoidal Model



The function above has a half-period of T and a maximum at $(T, 100)$. Using this information, the exact function shown above in Figure 4 is

$L(\Delta p) = \ 48.5\sin(\frac{\pi}{200}(\Delta p \ - 100)) + 51.5.$

Logistic Model:

Conventionally, Velhurst's logistic model models the population growth of a species with respect to finite resources and change in time. Thus, the population growth rate decreases when P (population) approaches the carrying capacity, K, of the environment, and the range of the function is constrained at K.Although the logistic model is predominantly applied in ecology and biology, if we consider the proportion of shares to be liquidated, $L(\Delta p)$, to be a population that has a limit (carrying capacity) of 100%with respect toΔp (which serves as the independent

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variable, similar to time in the original model), then the logistic model may be adapted. The adaptation to $L(\Delta p)$ would take the form of the following differential equation:

$$\frac{dL}{d\Delta p} = rL\left(1 - \frac{L}{100}\right) \tag{19}$$

where r controls the rate of change of $L(\Delta p)$with respect to Δp. $L = 100$ as that is the maximum % of the asset that can be liquidated – the constraint as $L(\Delta p)$ approaches the limit at 100. Integrating by partial fractions,[3] $L(\Delta p)$ can be expressed by the following function:

$$L(\Delta p) = \frac{100}{1 + Ce^{-r\Delta p}} \tag{20}$$

where $C = \frac{100}{m} - m$ as $L(0) = m$ as per the solution of the differential equation (15). r can be calculated by solving $L(T) = 99.9$ as itis the closest value to 100 that returns a single value of r. Applying the following rearrangements:

$$99.9 = \frac{100}{1 + Ce^{-rT}} \tag{21}$$

$$Ce^{-rT} = \frac{100}{99} - 1$$

$$e^{-rT} = \frac{0.009}{C}$$

$$-rT = \ln\left(\frac{0.009}{C}\right)$$

$$r = \frac{-\ln\left(\frac{0.009}{C}\right)}{T}$$

After substituting C,the complete model can be expressed as:

[3] Stewart, James. *CALCULUS : Early Transcendentals.* S.L., Cengage Learning, 23 Jan. 2020. The solution of the differential equation is adapted from the same.

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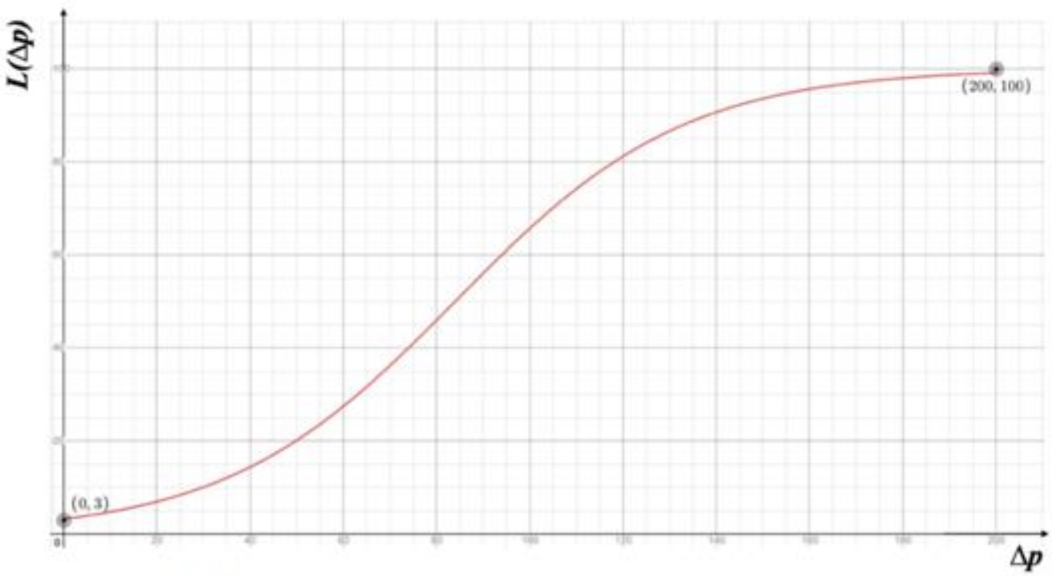
$$L(\Delta p) = \frac{100}{1 + \left(\frac{100}{m} - m\right) e^{\frac{\ln\left(\frac{0.009}{\left(\frac{100}{m} - m\right)}\right)}{T}\Delta p}} \tag{22}$$

Figure 5 displays $L(\Delta p)$ when $m = 3$ and $T = 200$.

Figure 5: The Logistic Model



At larger values of Δp, $L(\Delta p)$ approaches 100 with a y-intercept of (0,3). The exact function in Figure 5 (displayed to 3 significant figures) is $L(\Delta p) = \frac{100}{1+(30.3)e^{-0.0406\Delta p}}$.

Exponential Model:

Perhaps the most intuitive approach to $L(\Delta p)$ is an exponential relationship. The proportion liquidated increases exponentially with Δp in order to maximise profit. An exponential relationship is written as

$$L(\Delta p) = k \times b^{\Delta p} \tag{23}$$

Substituting Δp as 0,

$$L(0) = m$$

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$$k \times b^0 = m \tag{23}$$

$$k = m$$

We know that $L(T) = 100$, hence we can solve for b accordingly:

$$m \times b^T = 100 \tag{24}$$

$$b^T = \frac{100}{m}$$

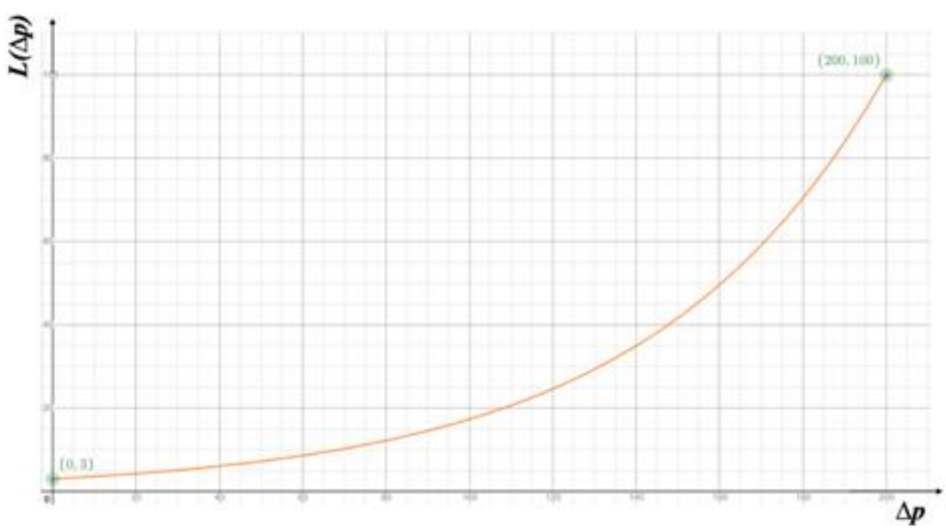
$$b = \sqrt[T]{\frac{100}{m}}$$

Hence, the final exponential model is:

$$L(\Delta p) = m\left(\sqrt[T]{\frac{100}{m}}\right)^{\Delta p} \tag{25}$$

Figure 6 shows $L(\Delta p)$ when $m = 3$ and $T = 200$.

Figure 6: The Exponential Model



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The exact function shown in Figure 6 is $L(\Delta p) = 3\left(\sqrt[200]{33.33}\right)^{\Delta p}$ for the domain and range specified above.

Power Model:

The power model is unique for it can be extended to create a more accurate model that allows one to set the rate of change as per their preferences. For example, if a trader wished to liquidate 50%of the asset when $\Delta p = 110$, the new model would be devised based on the condition $L(110) = 50$. Since the rate of change of $L(\Delta p)$ increases asΔp increases, the rate of change may be expressed as a power model wherein the rate of change is proportional to Δpraised to a certain power, as written below.

$$L'(\Delta p) = k\Delta p^n \tag{26}$$

Integrating the equation above results in the general solution:

$$\int (k\Delta p^n) d\,\Delta p = \frac{k\Delta p^{(n+1)}}{(n+1)} + c \tag{27}$$

The general solution is found by $c = m$ as $L(0) = m$. Solving for k

$$\frac{kT^{(n+1)}}{(n+1)} + m = 100 \tag{28}$$

$$kT^{(n+1)} = (n+1)(100-m)$$

$$k = \frac{(n+1)(100-m)}{T^{(n+1)}}$$

Hence, the final expression for $L(\Delta p)$ is

$$L(\Delta p) = \frac{(100-m)(n+1)\Delta p^{(n+1)}}{T^{(n+1)}(n+1)} + m \tag{29}$$

The particular solution then varies based on the trader's risk undertaking and personal preferences. For example, if $m = 3$ and $T = 200$ and the trader wishes that $L(150) = 50$, then we have the equation:

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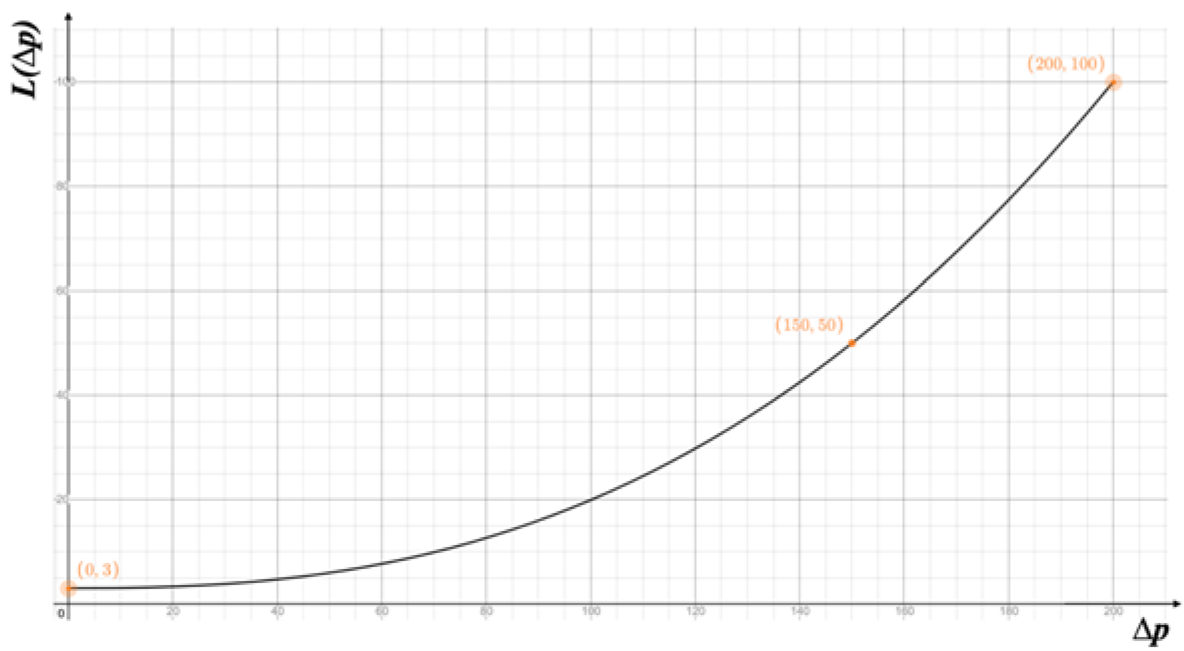
$$50 = \frac{97(n+1)150^{(n+1)}}{200^{(n+1)}(n+1)} + 3 \tag{30}$$

Solving the equation above to three significant figures, $n = 1.52$. In this instance, the particular solution of the model is

$$L(\Delta p) = \frac{97 \times 2.52x^{2.52}}{200^{2.52} \times 2.52} \tag{31}$$

This figure is represented in Figure 7 below.

Figure 7: The Power Model



Using the values specified above, the exact power model is simply equation (31) for the domain and range established above.

Evaluation of Models

In order to evaluate the models above, it is worth noting how values of $L(\Delta p)$ impact marginal revenue per trade. This is essential in understanding how revenue is maximised and generated.

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Let the volume of shares remaining after trade be v_t.Assuming that $L(\Delta p)$ is a %,the volume of shares remaining after trade 1can be represented as:

$$v_1 = V\big(1 - L(\Delta p_1)\big) \tag{32}$$

For trade 2, the remaining number of shares before the trade is v_1 in place of V.

$$v_2 = V_1\big(1 - L(\Delta p_2)\big) \tag{33}$$

$$v_2 = V\big(1 - L(\Delta p_1)\big)\big(1 - L(\Delta p_2)\big)$$

From the above equation, we may generalise the formula of v_t as:

$$v_t = V\big(1 - L(\Delta p_1)\big)\big(1 - L(\Delta p_2)\big)\big(1 - L(\Delta p_3)\big)\ldots\big(1 - L(\Delta p_t)\big) \tag{34}$$

However, v_t will always approach0 as all units of the asset that comprise of its total volume are liquidated at the final trade. Though this is intuitive, the consideration of revenue introduces an interesting perspective. Let S_t represent the total volume of the asset that has been sold at trade number t. The total number of units sold can be understood as the volume of asset remaining after trade t subtracted from the volume of the total asset. Modifying (30), S_t can be written as

$$S_t = V - \Big(V\big(1 - L(\Delta p_1)\big)\big(1 - L(\Delta p_2)\big)\big(1 - L(\Delta p_3)\big)\ldots\big(1 - L(\Delta p_t)\big)\Big) \tag{35}$$

This brings us to the idea of marginal revenue, which is the amount of revenue made

per each additional trade. The marginal number of shares liquidated at t iss_t. The mathematical expression is as follows:

$$s_t = S_t - S_{t-1} \tag{36}$$

Therefore, marginal revenue trade t is given by the multiplication below

$$R_m = \Delta p_t \times s_t \tag{37}$$

The total revenue earned after all trades have been made (v_t approaches 0) is

$$R = \sum_{t=1}^{t_{max}} (\Delta p_t \times s_t) \tag{38}$$

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where t_{max} is the trade at which $v_t$reaches 0. In order to maximise$R$, R_m needs to be maximised at every trade. As per the formulae of s_t and S_t, R_m increases as both Δp_t and s_t increase. We know that the higher the value of $L(\Delta p_t)$, the greater the value of s_t. To ensure that both Δp_t and s_t are at the highest possible values,$L(\Delta p_t)$should constantly increase as Δp_t approaches T.The rate of change of $L(\Delta p)$with respect to t must increase as Δp increases. Hence, we arrive at the criteria used to evaluate the following models.

$$L'(\Delta p) \textit{must continuously increase as} \Delta p \rightarrow T \textit{ergo reach its maximum at the point} \\ (T, 100) \textit{for} \{\Delta p |\ 0 < \Delta p \leq T | \Delta p \in R\}.$$

Finding the points of inflection for the models is useful in order to find the point where the derivative of $L(\Delta p)$ is maximised. A point of inflection is a point at which the tangent to the curve crosses the curve[4]. For $L(\Delta p)$, there is a point of inflection at $\Delta p = u$ if $L''(u) = 0$ and the sign of $L''(\Delta p)$ changes at $\Delta p = u$.Thus, the point of inflection tells us where the first derivative of the function is maximum which is critical for evaluating models as per the stipulated criteria.

Linear Model:

The rate of change of the linear model is simply the slope of the model. In our case, the first derivative is simply the value of k calculated in (7).So, the rate of change is

$$L'(\Delta p) = \frac{(100 - m)}{T} \tag{39}$$

However, $L'(\Delta p)$ is a constant in this model. This does not fit the criterion stipulated above as $L'(\Delta p)$does not increase as $\Delta p \rightarrow T$ nor does it have a maximum at$(T, 100)$. Thus, the linear model is unsuitable for maximising revenue in comparison to the other models.

Quadratic Model:

To find the rate of change, differentiate the quadratic model:

$$\frac{d}{d\Delta p}\left(\frac{(100 - m)}{-T^2}\Delta p^2 + \frac{2(100 - m)}{T}\Delta p + m\right) \tag{40}$$

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[4]Haese, Michael, et al. *Mathematics : Applications and Interpretation HL. 2 : For Use with IB Diploma Programme*. Marleston, Sa, Haese Mathematics, 2019.

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$$L'(\Delta p) = \frac{2(m-100)(\Delta p - T)}{T^2} \tag{41}$$

In this instance, $L'(\Delta p)$ is a continuously decreasing linear equation, which makes calculating the point of inflection redundant. Since the derivative never increases, $L'(\Delta p)$ does not increase as Δp reaches T. As the derivative has no maximum value, the quadratic model fails to satisfy the criterion.

Sinusoidal Model:

To find the rate of change of the sinusoidal model, the following derivative is utilised:

$$\frac{d}{d\Delta p}\left(\frac{(100-m)}{2}\sin(\frac{\pi}{T}(x \ -\frac{T}{2})) + \frac{(100+m)}{2}\right) \tag{42}$$

$$L'(\Delta p) = -\frac{\pi(m-100)\sin\left(\frac{\pi\Delta p}{T}\right)}{2T} \tag{43}$$

Note that for the model to be valid, $L'(\Delta p)$ must continuously increase or have a maximum at $\Delta p = T$. To find the maximum of the sinusoidal function, first the second derivative needs to be obtained.

$$L''(\Delta p) = -\frac{\pi^2(m-100)\cos\left(\frac{\pi\Delta p}{T}\right)}{2T^2} \tag{44}$$

Equating $L''(\Delta p)$ to 0,$\cos\left(\frac{\pi\Delta p}{T}\right) = 0$

$$\frac{\pi\Delta p}{T} = \cos^{-1}(0)\,\{\Delta p|\ 0 < \Delta p \leq T|\Delta p \in Q\} \tag{45}$$

$$\pi\Delta p = \frac{\pi \times T}{2}$$

$$\Delta p = \frac{T}{2}$$

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At $\Delta p = \frac{T}{2}$, there is a sign change in the second derivative from positive to negative which shows that $\Delta p = \frac{T}{2}$ is a point of inflection for the sinusoidal model of L(Δp). It is at this point at which the derivative has a local maxima for the domain $\{\Delta p|\ 0 < \Delta p \leq T|\Delta p \in Q\}$.

$As\ L'(\Delta p)$ must always be increasing, the sinusoidal model partially suits the criterion for $\left\{\Delta p \,\middle|\, 0 < \Delta p \leq \frac{T}{2} \middle|\, \Delta p \in Q\right\}$. However, for $\left\{\Delta p \,\middle|\frac{T}{2} < \Delta p \leq T \,\middle|\, \Delta p \in Q\right\}$, the model fails to satisfy the criterion. Thus, the sinusoidal model cannot maximise marginal revenue for higher values of Δp and is therefore unsuitable as a general model for partial liquidation.

Logistic model:

The first derivative of the aforementioned logistic function can be represented in the following manner.

$$\frac{d}{d\Delta p}\left(\frac{100}{1+\left(\frac{100}{m}-m\right)e^{\frac{\ln\left(\frac{0.009}{\left(\frac{100}{m}-m\right)}\right)}{T}\Delta p}}\right) \tag{46}$$

$$L'(\Delta p) = -\frac{100\ln\left(\frac{9}{1000\left(\frac{100}{m}-m\right)}\right)\left(\frac{100}{m}-m\right)e^{\frac{\ln\left(\frac{9}{1000\left(\frac{100}{m}-m\right)}\right)\Delta p}{T}}}{T\left(\left(\frac{100}{m}-m\right)e^{\frac{\ln\left(\frac{9}{1000\left(\frac{100}{m}-m\right)}\right)\Delta p}{T}}\right)^2} \tag{47}$$

The second derivative must be found to calculate the point at which $\frac{dL}{d(\Delta p)}$ is maximised. The expression for the second derivative is summarised below.

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$$L''(\Delta p) = -\frac{100\ln\left(\left(\frac{9}{1000\left(\frac{100}{m}-m\right)}\right)\right)^2 m \times (m^2-100)\, e^{\frac{\ln\left(\frac{9}{1000\left(\frac{100}{m}-m\right)}\right)\Delta p}{T}} \times \left((m^2-100)e^{\frac{\ln\left(\frac{9}{1000\left(\frac{100}{m}-m\right)}\right)\Delta p}{T}}+m\right)}{T^2\left((m^2-100)e^{\frac{\ln\left(\frac{9}{1000\left(\frac{100}{m}-m\right)}\right)\Delta p}{T}}-m\right)^3} \tag{48}$$

To find the point of inflection, we solve the equation below:

$$0 = -\frac{100\ln\left(\left(\frac{9}{1000\left(\frac{100}{m}-m\right)}\right)\right)^2 m \times (m^2-100)\, e^{\frac{\ln\left(\frac{9}{1000\left(\frac{100}{m}-m\right)}\right)\Delta p}{T}} \times \left((m^2-100)e^{\frac{\ln\left(\frac{9}{1000\left(\frac{100}{m}-m\right)}\right)\Delta p}{T}}+m\right)}{T^2\left((m^2-100)e^{\frac{\ln\left(\frac{9}{1000\left(\frac{100}{m}-m\right)}\right)\Delta p}{T}}-m\right)^3} \tag{49}$$

While the equation above can be equated to 0 with the help of technology, this process can be complicated and varies with the value of m and T. It can be noted that for $m < 10, L'(\Delta p)$ reaches its maximum for values of $\Delta p < \frac{T}{2}$.If we analyse the graph of the logistic model,$L'(\Delta p)$is low for lower values of Δp and increases temporarily. However, for values of Δp greater than the maximum point, $L'(\Delta p) \to 0$ as $\Delta p \to T$, which makes the logistic model unsuitable for maximum revenue generation.

The Exponential and Power Model:

If $L(\Delta p)$ is an exponential function, $L'(\Delta p)$is also exponential. Subsequently, there is no local maxima, only an increase in $L'(\Delta p)$ as Δp approaches T. $L'(\Delta p)$varies heavily based on the values of m and T. The expression of the first derivative of the exponential model is given below:

$$L'(\Delta p) = \frac{-m(\ln(m) - \ln(100)) \times 100^{\frac{\Delta p}{T}}}{T \times \left(m^{\frac{1}{T}}\right)^{\Delta p}} \tag{50}$$

The second derivative of the function is as follows:

$$L''(\Delta p) = \frac{m(\ln(m) - \ln(100))^2 \times 100^{\frac{\Delta p}{T}}}{T^2\left(m^{\frac{1}{T}}\right)^x} \tag{51}$$

The expression above will never equal 0 for $\Delta p > 0$. The same reasoning holds true for the power model in (29). The first derivative of(29)can be written as

$$L'(\Delta p) = \frac{(100 - m)(n + 1)\Delta p^n}{T^{n+1}} \tag{52}$$

The second derivative of the same is

$$L''(\Delta p) = T^{(-n-1)}(100 - m)n(n + 1)x^{(n-1)} \tag{53}$$

Similar to the previous derivative, this will also never equate to 0 for the domain of $L(\Delta p)$. Note that for both these exponential functions, the derivatives reach their local maximums at the point $(T, 100)$. Thus, s_t always increases at an increasing rate with respect to Δp_t which allows for revenue maximisation and when Δp_t is low, Δs_t is also low along with marginal revenue. However, as the change in v_t is also low, it leaves a larger volume to liquidate when Δp_t reaches higher prices. The same occurrence happens to an extent at stage Y of the logistic model, but given the nature of exponential increase, revenue is maximised by exponential movements of $L(\Delta p)$.

Upon further analysis, it must be noted that $n > 1$ at the very least for the exponential model to have a non-constant derivative. In general, the higher the value of n, the greater $L'(\Delta p)$ as $\Delta p \to T$, and the lower Δp values are, the lower the rate of change. To maximise the revenue generated by the power model compared to the exponential model, the derivative of the power model at $\Delta p = T$ must be greater than the derivative of the exponential model. This adheres to the criterion as the derivative of the power model is both increasing and higher than the exponential model when$\Delta p \to T$. By substituting $\Delta p = T$ into (50) and (52), we solve the following inequality for n using technology.

$$\frac{(100 - m)(n + 1)T^n}{T^{n+1}} > -\frac{m\,(\ln(m) - \ln(100)) \times 100}{T \times \left(m^{\frac{1}{T}}\right)^T} \tag{54}$$

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$$\frac{(100-m)(n+1)T^n}{T^{n+1}} > -\frac{(\ln(m)-\ln(100)) \times 100}{T} \tag{55}$$

For analytical purposes, let the inequality above simplify to $n > u$ such that $u > 2$. As n increases, the actual Δp at which

$$\frac{(100-m)(n+1)\Delta p^n}{T^{n+1}} > \frac{-m(\ln(m)-\ln(100)) \times 100^{\left(\frac{\Delta p}{T}\right)}}{T \times \left(m^{\frac{1}{T}}\right)^{\Delta p}} \tag{56}$$

increases: the larger the value of n, the value of $\frac{(100-m)(n+1)\Delta p^n}{T^{n+1}}$is lower for smaller values of Δp. By virtue of the derivatives, we may conclude that the **general equation** for the model that maximises revenue is:

$$L(\Delta p) = \frac{(100-m)(n+1)\Delta p^{(n+1)}}{T^{(n+1)}(n+1)} + m \; such$$
$$thatn \;(constant\ acting\ as\ an\ exponent)\ isgreater\ than\ u\ whereu\ is\ the\ minimum\ value\ of\ n$$
$$at\ which\ L'(\Delta p) = \frac{-(ln(m)-ln(100)) \times 100}{T}$$

Thus, the exponential model is the most suitable model for partial liquidation. This conclusion is also quite logical as one should sell more at higher prices compared to lower prices and the greater the amount sold at higher prices, the greater the revenue thus volume sold should rise exponentially with respect to increases in Δp in relation to T.

Conclusion: To conclude, a variety of mathematical models can be used to model a relationship between the proportion of a tradeable asset liquidated and the increase in the price of the asset. However, the most optimum relationship between $L(\Delta p)$ and Δp is exponential as in an exponential relationship, the derivative is maximised at the highest value of Δp. The value of n becomes the principal determinant for the amount liquidated and must be set such that the first derivative of the extended model is greater than that of the exponential model. The ideal model as per the assumptions and conditions established throughout the investigation is:

$$L(\Delta p) = \frac{(100-m)(n+1)\Delta p^{(n+1)}}{T^{(n+1)}(n+1)} + m \; such$$
$$thatn \;(constant\ acting\ as\ an\ exponent) is\ greater\ than\ u\ where\ u\ is\ the\ minimum\ value\ of$$
$$n\ at\ which\ L'(\Delta p) = \frac{-(ln(m)-ln(100) \times 100}{T}$$

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As far as risk is involved, a differential equation can be enhanced using technology to give a particular solution that considers risk. Given that the model only treats positive changes in prices which are less than a threshold value based on higher past price values, the model is best suited for short-term swing-trading or scalping as these are strategies that usually capitalise on relatively small, short term price increases; these situations resemble the domain of $L(\Delta p)$.

Limitations:

One of the main limitations of this modelling process is that it assumes that Δp is always less than T. Although T is rooted in previous values of the asset's price, the model cannot account for situations where Δp is greater than P. Similarly, if p_0 is the singular highest price of the asset, then the modelling process fails as T cannot be calculated. Moreover, besides the exponential function, at values where Δp approaches T, the derivatives of all the derived functions are close to 0 especially at higher values of T (like when T is above 1000). This implies that the absolute rates of change are quite low which may be limiting to the maximisation of revenue for low values of Δp.

With different applications, more modifications would arise which would further optimise the model. Given that $L(\Delta p)$ is a proportion, the model would be best suited for currencies, cryptocurrencies, and other tradeable assets where it is possible to liquidate proportions. For example, half a share cannot be traded. The models listed above are fixed depending on T and m. The only model that offers any type of flexibility is (29), which through integration allows for the consideration of risk that a trader undertakes based on their own thresholds and values of Δp at which a preferred % of V can be liquidated. Given the low derivative of this model for values of Δp, s_t is quite low which means that for low values of Δp, $L(\Delta p)$ is relatively low. Similarly, s_t may be less than mV. This would result in R being maximised only at high values of t. So, unless Δp is near T, the exponential model generates less revenue. Depending on the values of m and T, the sinusoidal and logistic model may be appropriate. In general, however, the power is a better model as it has the potential for modification and is high yielding as $\Delta p \rightarrow T$.

Moreover, traders may be deterred by the loss aversion bias as per behavioural economics. The loss aversion bias refers to the phenomenon in which humans are psychologically impacted more by loss as opposed to gain; this prompts deviations from rational decision making to decisions that minimise loss rather than maximising gain. If R_m is low, revenues may be lower, and traders may experience a sense of loss before S_t approaches V which may lead to the trader deviating from the modelling process. Similarly, s_t may be less than mV due to the low first derivative which makes trading unfeasible as in this instance, the amount liquidated is below the minimum amount that must be liquidated in a particular trade. In addition, this model assumes continuous liquidation. The only condition to consider liquidation is that $\Delta p > 0$. This means that at some higher values of Δp, revenue earned may be lower due to the amount liquidated in previous trades. Lastly, $L(\Delta p)$ may also be more maximising if $L(\Delta p)$ is linked to the probability ofΔp being in a particular range of values. The higher the probability of Δp being at a particular value, the higher the amount liquidated. This model may be more realistic considering values of Δp at a particular time and moment.

Future Scope:

With further testing involving real-world assets and values of T and m, the overall liquidation process can be refined. An interesting angle to research could be relating $L(\Delta p)$ to different probabilities of an asset changing by particular Δp values. Markov Chains and probability distributions could be applied to the same. This would result in a more relevant function as it would be rooted in probable values of Δp rather than optimisation through rates of change. All in all, this investigation exemplifies how standard mathematical functions have applications in the financial world and how different models and approaches interact to produce a method for maximising gain.

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