

Cognitive Structure Analysis: A Technique For Assessing What Students Know, Not Just How They Perform

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ABSTRACT

Assessment has been a key part of education, playing the role of determining how much students have learned. Traditionally, assessments have focused on whether students give the correct answer to problems, implying that the number of correctly-answered test items is a valid measure of how much students know. Unfortunately, the focus on correct answers has also resulted in neglecting the potential ability of assessments to provide diagnostic feedback to educators as to what concepts students have mastered and where the gaps in their knowledge are, thus potentially informing the day-to-day teaching process. The present paper describes an assessment technique called Cognitive Structure Analysis that is derived from John Leddo's integrated knowledge structure framework (Leddo et al., 1990) that combines several prominent knowledge representation frameworks in cognitive psychology. Using a Google Form, students from Pakistan were queried on four types of knowledge considered the basis of mastery of Algebra 1 concepts: factual, procedural, strategic, and rationale. From students' responses to these queries, measures of each type of knowledge and a combined knowledge score were created. Students were also given problems to solve. Correlations between each knowledge component score and problem-solving performance were high and the correlation between overall CSA-assessed knowledge and problem-solving performance was .86. Results suggest that CSA can be both easily implemented and highly diagnostic of student learning needs. Future research can investigate CSA's robustness across other subjects and whether incorporating CSA as part of day-to-day classroom instruction can lead to higher student achievement.

INTRODUCTION

Assessment has long been an integral part of the education process. It is seen as the measurement of how much students have learned the content that they were taught. In both classroom settings and in standardized testing, "learned the content" is typically operationally defined in terms of

the number of correct answers a student gives on test questions. Indeed, classical test theory, one of the major pillars of assessment methodology, assumes that the total number of correctly-answered test items indicates the students' level of knowledge (cf., de Ayala, 2009).

Over the years, a number of assessment frameworks have been utilized by teachers and educational organizations. Typically, these can be categorized by whether students are asked to select the correct answer from a set of answer choices or asked to construct their own answers to problems. There has been considerable debate over which category of method is better, with pros and cons attached to each. Multiple choice tests require students to select answers from several distracters. Multiple choice tests are widely used in standardized testing and in many classroom settings due to the ease of grading (Chaoui, 2011) and the fact that students often score higher on multiple choice tests than they do on constructive response tests as students can increase their scores through guessing (cf. Elbrink and Waits, 1970; O'Neil and Brown, 1997). However, such guessing is often cited by critics as a reason why multiple choice tests should not be used.

At the other end of the continuum are constructive tests, which require that students enter answers to questions rather than choose from different answer choices. Researchers find, when investigating math problem solving, that students are more likely to use guessing strategies when given multiple choice tests but are more likely to reason mathematically when given constructive tests (Herman et al., 1994), thus making the test more ecologically valid in measuring students' actual knowledge (Frary, 1985).

The challenge with the key assumption of classical test theory, that correct answers indicate learning and vice versa, is that this assumption may not be entirely true. A medical analogy works well here. Normally, if a person shows outward signs of illness, s/he is probably sick (although there could be non-medical reasons why a person can appear sick such as overexertion or lack of sleep). Similarly, a student who makes a lot of mistakes on a test probably has a lack of knowledge (unless, for example, s/he was distracted or sick during the test). However, a person can look healthy and still have an underlying illness. Similarly, a student may get correct answers on a test and have knowledge deficiencies. S/he can be parroting facts or formulas that s/he does not really understand or guessing correctly on multiple choice exams (which is a major criticism of that testing format). More importantly, the lack of correct answers does not inform the teacher as to what concepts need to be remediated. A doctor does not stop his/her diagnosis after observing symptoms. The doctor runs further tests to discover the cause of the symptoms, so that an appropriate remedy can be applied. Indeed, we would consider it medical malpractice for a doctor to treat only the symptoms and not the underlying causes of diseases. Similarly, an incorrect answer to a test question is a symptom that may indicate an underlying knowledge deficiency. We consider it to be educational malpractice to stop the assessment at that point

without diagnosing the underlying knowledge deficiency that is causing that incorrect answer. Unless that cause is identified, how can the appropriate remedial instruction be given?

The present paper reports an assessment methodology called Cognitive Structure Analysis (CSA) that is designed to assess the underlying concepts a student has, so that when a student does make a mistake, the source of that mistake can be identified and remediated. CSA is based on decades of cognitive psychology research that have shown that people possess a variety of knowledge types, each of which is organized and used differently in problem solving. Because there are different types of knowledge that people have, our framework is an integration of several prominent and well-researched formalisms. These include: semantic nets, which organize factual information (Quillian, 1966); production rules, which organize concrete procedures (Newell and Simon, 1972); scripts, which are general goal-based problem solving strategies (Schank and Abelson, 1977; Schank, 1982); and mental models, which explain the causal principle behind concepts (de Kleer and Brown, 1981). Because our framework integrates these four knowledge types, it is called INKS for INtegrated Knowledge Structure. We note that the National Council of Teachers of Mathematics (2000) has developed a taxonomy of strands necessary for students to be considered mathematically proficient that uses similar terminology: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning. In many ways, the strands of conceptual, procedural and strategic do correspond to our own. The key difference is that the National Council of Teachers of Mathematics frames these strands in terms of desired skills/behavioral outcomes whereas we conceptualize these in terms of the specific knowledge needed to achieve those outcomes.

The INKS framework is based on research by John Leddo (Leddo et al., 1990) that shows that true expertise in a subject area requires all four knowledge types. INKS also has implications for instruction. For example, in John Anderson's ACT-R framework, people initially learn factual/semantic knowledge that is later operationalized into procedures (Anderson, 1982). Research by Leddo takes this one step further showing that expert knowledge is organized around goals and plans (referred to in the literature as "scripts" – Schank and Abelson, 1977; Schank, 1982) and abstracted into causal principles (referred to in the literature as "mental models" – cf., de Kleer and Brown, 1981) that allow people to construct explanations and make predictions/innovations in novel situations. In order to identify the root cause of the mistake, we use a query-based assessment framework called Cognitive Structure Analysis (CSA, Leddo et al., 1990). CSA incorporates principles from the INKS knowledge representation framework. CSA is chosen because previous research shows that there is a strong correlation between user knowledge as assessed by CSA and performance in practical problem solving. In one previous research project, we found that the using an automated multiple-choice CSA system to assess student learning produced measures of knowledge that correlated .88 with student problem

solving performance and measures of change of knowledge as a result of instruction that correlated .78 with change in performance from pre-test to post-test. Moreover, at-risk students taught based on the needs diagnosed using CSA performed at a mainstream level three grades higher than their own after a 25-hour tutoring program in science (Leddo and Sak, 1994). In two other projects, assessments produced using the CSA methodology produced assessments of student learning agreed with teachers' assessments approximately 95%-97% of the time, which was statistically equal to teachers assessments with each other (Leddo et al., 1998, Liang and Leddo, 2021).

In a precursor project (Leddo et al., 2022), the present project, topics in Algebra 1 were analyzed for the facts (semantic knowledge), strategies (scripts), procedures (production rules) and rationales behind the concepts (mental models). Open-ended questions were then constructed to see if students possessed each of these knowledge components. Students' answers to the questions were analyzed for their correctness. This allowed the researchers to obtain knowledge scores for each of the four types of knowledge that are considered necessary to master an algebra topic as well as an overall knowledge score. These scores were then correlated with students' performance in problem solving to see how well they predicted that performance. Results of that experiment showed that correlations between overall problem solving performance and overall INKS knowledge was .966, between performance and factual knowledge was .866, between performance and procedural knowledge was .937, between overall performance and strategic knowledge was .819, and between overall performance and rationale knowledge was .788. The combined and individual correlations show CSA can be a powerful tool in assessing what students know. The purpose of the present study is to replicate the Leddo et al. (2022) in another country, in this case, Pakistan. This can help determine the universal applicability of CSA as an assessment tool.

METHOD

Participants

Participants were 18 middle and high school students in Pakistan. In order to get a range of knowledge levels across participants, students were recruited with a wide range of mathematical experience. At minimum, students were at least already taking Algebra 1, but may not have completed it.

Materials

There were two assessments created: a knowledge assessment that used CSA to assess the INKS-based knowledge categories of facts, strategies, procedures, and rationales; a post-test problems solving assessment that asked students to solve problems based on the topics covered

in the experiment. One mathematics assignment was created covering topics from Algebra 1. Topics and questions were chosen from online math textbooks created by leading American publishers.

The subjects for the assignments were: linear equations with variables on both sides, linear equations with unknown coefficients, compound inequalities, solving equations with square roots, and quadratic formulas. The assessment was composed of three sections. The first section included fact-based problems to examine students' understanding of the material's definitions which were in total 9 questions. The next section was composed of strategy based problems, procedure based problems and rationale based problems. The strategy based problems were composed of 5 questions, the procedure based problems were composed of 8 questions and the rationale based problems were composed of 5 questions. The last section of the form contained Post test problems which were a total of 25 math problems the participants had to solve. This form contained multiple Algebra 1 topics which consisted of Equations, Exponents, Polynomials and Inequalities.

Facts Based Questions:

"What is a variable? How is it represented in a problem?"

"What is a square root?"

"What do you need to pay attention to when solving an inequality question? What are some of the differences between inequality and equation problems?"

"What is a coefficient?" "Where do you find the coefficient in a problem?"

"What is a radical?"

"What is the standard form of the quadratic equation?"

"What is the difference between "and" and "or" when solving inequality problems?"

"What is a constant"? "Where do you find the constant in a problem?"

"What is the relationship between a radical and square root?" "How do you identify a square root in a problem?"

Strategies:

Solve for f:

$$-f + 2 + 4f = 8 - 3f$$

Solve for y:

$$a * (n + y) = 10y + 32$$

Solve compound inequalities:

$$2x + 3 \geq 7 \text{ OR } 2x + 9 > 112$$

Solve for x:

$$\sqrt{(2x - 1)} = 7$$

Solve for s:

$$s^2 - 2s - 35 = 0$$

Procedures:

“How do you identify a coefficient in a linear equation with unknown coefficients?”

“How do you combine like terms?”

“How do you remove the constants to one side of the equation and variables to the other side of the equation?”

“How to isolate the variable in a linear equation with variables on both sides?”

“How do you perform additive inverse and multiplicative inverse when a negative number is involved? Are there any changes to the signs?”

“How do you represent your set of answers on the graph when solving compound inequalities questions?”

“How do you determine what is your final answer in “and”/“or” condition when solving compound inequalities questions?”

“How do you isolate the radical term in an equation?”

Rationales:

“If you subtract a number on one side of the equation, how do you perform it on the other side of the equation?”

“Why do you need to keep both sides equal when solving linear equations? Why is it important to keep the equation balanced?”

“Why is it important to keep variables on one side of the equation, and keep constant and unknown coefficients on the other side?”

“Why do you need to express the final result based on two solutions when solving compound inequalities questions?”

“Why do you need to square both sides of the equation to solve a radical problem?”

“Why should you simplify the equation after eliminating the radical?”

”Under what conditions will you only acquire one final result, and under what conditions will you have a set of results when solving compound inequalities questions?”

Post-test Problems:

Solve for n:

$$-6n - 20 = -2n + 4(1 - 3n)$$

Simplify:

$$(3x - 4)^2$$

Solve compound inequalities

$$1 - 4x < 21 \text{ and } 5x + 2 > 22$$

Simplify:

$$\sqrt{5}(2\sqrt{3} + \sqrt{12})$$

Solve compound inequalities:

$$2x + 7 < -11 \text{ or } -3x - 2 > 13$$

Solve for a:

$$7(5a - 4) - 1 = 14 - 8a$$

Solve for y:

$$12y + d = -19y + t$$

Solve x using quadratic formula:

$$z(z-1) = 3$$

Solve x using quadratic formula:

$$2(w^2 - 2w) = 5$$

Solve for y:

$$v(j + y) = 61y + 82$$

Solve for x:

$$-3(x - 1) + 8(x - 3) = 6x + 7 - 5x$$

Simplify:

$$\sqrt{(45 * 9 + 27 * 36)}$$

Solve for x:

$$-31 - 4x = -5 - 5(1 + 5x)$$

Solve compound inequalities:

$$3x - 2 > -8 \text{ or } 2x + 1 < 9$$

Solve x using quadratic formula:

$$x^2 = 3x - 1$$

Solve x using quadratic formula:

$$x^2 = 3x - 1$$

Solve x using quadratic formula:

$$x^2 - 11x + 28 = 0$$

Solve compound inequalities:

$$4x - 2 > 10 \text{ and } 3x + 1 < 22$$

Solve for w:

$$\sqrt{(8w + 9)} = w$$

Solve x using quadratic formula:

$$5x^2 + 6x + 1 = 0$$

Solve for n:

$$-8n + 4(1 + 5n) = -6n - 14$$

Solve for x:

$$ax + 3x = bx + 5$$

Solve compound inequalities:

$$2y + 7 < 13 \text{ or } -3y - 2 < 10$$

Solve for x:

$$\sqrt{(2x-5)} - 3 = 1 - x$$

Solve for x:

$$a(5 - x) = bx - 8$$

Procedure

The materials were administered in the form of a Google Forms survey. Participants were given links to the survey and asked to fill out the survey. They were given as much time as needed. No calculators or outside resources were allowed. Participants were supervised to prevent any use of outside resources.

RESULTS

The math facts section had a total of 9 questions. The math procedures section had a total of 8 questions. The math strategies section had a total of 5 questions and the math rationales section also had a total of 5 questions. Students' answers to the knowledge questions were graded on an all or none basis, depending on whether they substantially matched the definitions from the sources from which they were taken. Therefore, a total score was calculated for each student for

how many fact, strategy, procedure, and rationale questions they correctly answered. The problem-solving questions were scored based on whether the students gave the correct answer. Therefore, each student could get up to 25 points.

From there, correlation coefficients were calculated between the total number of problem-solving questions each student correctly answered and the fact, strategy, procedure, rationale and combined (all four knowledge categories added together) scores. Table 1 below shows the correlation coefficients between total problems correctly answered on the post-test and total and individual INKS knowledge type questions correctly answered. For comparison purposes, the correlation coefficients for the Leddo et al. (2022) study are also shown.

Table 1: Correlations between post-test problem solving and CSA-assessed INKS knowledge.

	Total knowledge	Factual knowledge	Procedural knowledge	Strategic knowledge	Rationale knowledge
Pakistan	.86	.956	.965	.632	.888
United States	.966	.866	.937	.819	.788

The above table shows that, as with the Leddo et al. (2022) study, both overall and individual component INKS knowledge types correlate highly with post-test problem solving performance, all p values < .01. There are some minor differences in the data collected from US and Pakistani students. Overall, CSA was somewhat more predictive of problem solving performance in US students than in Pakistani ones (.966 vs .86). Interestingly, both factual and procedural knowledge had the highest predictive power in both countries' students. The other notable difference between the two countries' students is that US students showed a stronger correlation with strategic knowledge and Pakistani students showed a stronger correlation with rationale knowledge.

DISCUSSION

The results of the present project demonstrate the feasibility of using CSA as a method to assess how well students have learned algebraic topics. The correlations between the assessed individual knowledge components of the INKS framework and problem-solving performance were all high and the overall correlation between the assessed overall INKS knowledge and

problem solving was .86 in Pakistan and .966 in the US. The benefit of the CSA technique is its simplicity as well as its power. In the present study, CSA was implemented through a Google Forms survey. This suggests that it is readily scalable and can be implemented in educational settings with minimal disruption to existing practices and minimal training on the part of teachers. We definitely see such implementation as a logical next step of the present work. The present research still leaves interesting questions unanswered. The present CSA technique neither measures a student's tendency to make careless mistakes nor quantifies the impact of such mistakes on the final test score. This does seem to be a surmountable problem. In Liang and Leddo (2021), software was created to probe students' underlying INKS knowledge after they made mistakes. If the student demonstrated mastery of the knowledge but still made a mistake, the software labeled the mistake as a careless one. This can be incorporated into the present framework.

Another important research direction to take is to conduct a systematic replication of the above experiment across different math subjects (e.g., pre-algebra, geometry). Of interest is not only whether the basic predictive power of the INKS framework holds but also whether different types of knowledge need to be included or different correlational strengths will emerge. Algebra 1 is highly procedural, so it is not surprising that of the four categories of INKS knowledge, factual and procedural knowledge (the two most concrete types of knowledge) were the most individually predictive of overall problem solving. Geometric proofs are much more strategic in nature, so it may be the case that strategic or script-based knowledge will prove even more important for solving geometric proofs than it did for solving algebraic equations. Similarly, while algebra is highly symbolic in nature, geometry involves shapes and visual/spatial reasoning may play an important role, something not contained in the present CSA framework. CSA, and its theoretical basis INKS, may need to be expanded to incorporate this type of knowledge.

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