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Investigating Performance of ESN's in Forecasting Financial Metrics When Compared To Traditional RNN Types

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ABSTRACT

This research investigates the performance of Echo State Networks (ESN) in forecasting financial metrics and compares their effectiveness against traditional recurrent neural network (RNN) architectures like Long Short-Term Memory (LSTM) networks and Gated Recurrent Units (GRU), as well as Generalized Auto Regressive Conditional Heteroskedasticity (GARCH) models. By analyzing datasets sourced from Yahoo Finance for various financial indices, exchange-traded funds and stocks over five years, this study examines the accuracy, and structural simplicity of ESNs in predicting close prices, daily volatility, and log returns. Results indicate that ESNs, with their reservoir computing capabilities, outperform traditional RNNs by achieving lower mean absolute error (MAE) and mean squared error (MSE) overall, highlighting their potential as efficient and robust tools for financial time-series forecasting.

Introduction

In the fast-paced and complex world of financial markets, the accurate forecasting of stock prices, volatility, and other financial metrics is crucial for investors, analysts, and policymakers. These forecasts influence investment strategies, risk management, and market understanding. Traditionally, various models, including linear regressions and more sophisticated neural networks, have been employed to tackle the challenges of financial time-series forecasting. However, issues such as high computational costs, slow convergence rates, and the complexity of network architectures often undermine their efficacy.

The advent of Echo State Networks (ESN), a type of RNN under the umbrella of reservoir computing, offers a promising alternative. Known for their simple structures and efficient data

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processing[1], ESNs reduce the need for intensive computational resources while maintaining the ability to capture complex nonlinear relationships inherent in financial data, and ability to become universal approximators for dynamic systems like the stock market. This study provides a comparative analysis of ESNs against traditional RNNs (LSTM and GRU) and GARCH models, focusing on their application in forecasting key financial metrics like close prices, daily volatility, and log returns across various financial entities and time horizons.

Throughout the course of this investigation, ESNs demonstrated a significant advantage in predictive accuracy and operational efficiency, particularly in longer-term forecasting scenarios. Results revealed that ESNs achieved lower MSEs and MAEs compared to LSTM, GRU, and GARCH models overall, and outperformed their peers especially in close price prediction tasks. These findings underscore the potential of ESNs to act as a superior forecasting tool in financial markets, offering both speed and precision in a simplified architectural framework.

Literature Review

The use of Echo State Networks (ESNs) in financial modeling has seen significant advancements with both recent, and older studies demonstrating their potential to outperform traditional models in forecasting financial metrics like volatility and stock price. The first article from our literature review, published in 2021, introduces a novel hybrid model integrating the Echo State Network (ESN) with the Heterogeneous Autoregressive (HAR) model and Particle Swarm Optimization (PSO), referred to as HAR-PSO-ESN. This model leverages the quick adaptability of ESNs to capture complex patterns in time-series data while utilizing the feature design strengths of the HAR model and the optimization capabilities of PSO[2]. The study tested the model's effectiveness against traditional models like ARIMA and MLP using NASDAQ stock volatilities, demonstrating superior predictive performance with statistically significant improvements in R-squared and mean squared error across multiple forecasting horizons. The second article from 2014 explores the application of deterministic ESNs in stock price forecasting. Unlike typical ESNs that use randomly generated reservoirs, deterministic ESNs employ a pre-defined reservoir structure, which simplifies model construction and potentially enhances performance predictability and optimization. The study tested deterministic ESNs against standard ESNs

[1] Guang, Sun, et al. "Stock Price Forecasting: An Echo State Network Approach." *Semantic Scholar*, National Natural Science Foundation of China, 2021, pdfs.semanticscholar.org/e3a7/2a2b7d6071461007112f12c8716529755d39.pdf.

[2] Ribeiro, Gabriel Trierweiler et al. "Novel Hybrid Model Based on Echo State Neural Network Applied to the Prediction of Stock Price Return Volatility." *Science Direct*, National Council of Scientific and Technologic Development of Brazil, 29 June 2021,

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using the Shanghai Composite Index and S&P 500 datasets[3]. Results indicated that deterministic ESNs, with their simpler and more predictable reservoir construction, outperformed standard ESNs in terms of forecasting accuracy and computational efficiency. Building on Existing Work Our research builds on these studies by extending the application of ESNs to a broader array of financial metrics and market conditions.

While the first study successfully combined ESN with HAR and PSO to enhance volatility forecasting, our work explores a similar hybrid approach but extends the use of ESNs beyond just volatility to other financial metrics like close prices and log returns across different market indices and volatility levels. Moreover, our approach involves comparing the performance of ESNs not only with traditional models like GARCH and LSTM but also within different configurations of reservoir computing models, including the novel types of deterministic ESNs mentioned in the second study. This comparative analysis across a diverse set of financial metrics and model types offers deeper insights into the applicability and robustness of ESNs in financial forecasting, potentially paving the way for more targeted and effective financial decision-making tools.

Background Information

Echo State Networks (ESN)

Introduced by Herbert in 2001[4], the Echo State Network (ESN) is a recurrent neural network (RNN) variant that leverages a reservoir computing framework. Initially, ESNs were primarily theoretical constructs until their practical application in fields related to wireless communications began in 2004. Since then, substantial advancements have been made in refining the model. Traditional RNNs often suffered from instability, computational complexity, and slow convergence rates. In contrast, ESNs offer simpler computations, faster processing times, and shorter cycles, significantly enhancing time-series forecasting applications.

The typical structure of an ESN includes input, reservoir, and output layers, as depicted in Figure 1. This structural model comprises H input nodes, N reservoir processing neurons, and L output nodes.

[3] Fang, Bin. “Deterministic Echo State Networks Based Stock Price ...” *ResearchGate*, Fundamental Research Funds for the Central Universities in China, 2014, www.researchgate.net/publication/275468711_Deterministic_Echo_State_Networks_Based_Stock_Price_Forecasting.

[4] Jaeger, Herbert. “(PDF) the" Echo State" Approach to Analysing and Training Recurrent Neural Networks-with an Erratum Note’.” *ResearchGate*, 6 Dec. 2001, www.researchgate.net/publication/215385037_The_echo_state_approach_to_analysing_and_training_recurrent_neural_networks-with_an_erratum_note’.

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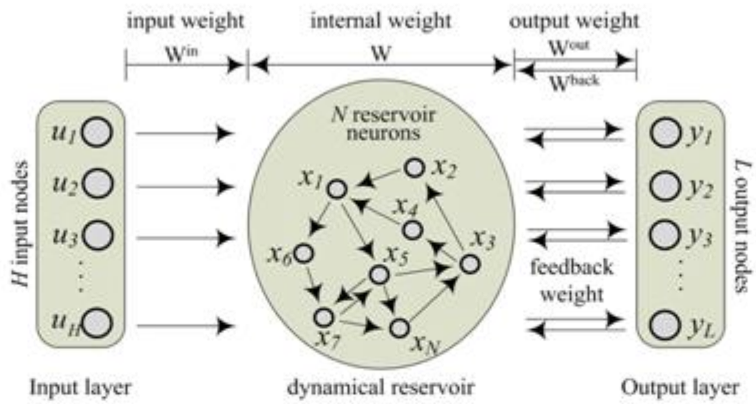
Figure 1: Barebones ESN Architecture[5]



The vectors for input stock data u, output prediction value y, and reservoir state space x are defined by their respective dimensions m, n, and p:

$$u(t) = (u_1(t),\dots,u_H(t))$$

$$y(t) = (y_1(t),\dots,y_L(t))$$

$$x(t) = (x_1(t),\dots,x_N(t))$$

The connections within the ESN are defined by several matrices:

- Input to reservoir weights (W_{in}): An N x K matrix.
- Feedback weights from the output to the reservoir (W_{back}): An N x L matrix.
- Internal connections within the reservoir (W): An N x N matrix.
- Output weights (W_{out}): An L x (K + N + L) matrix, connecting the reservoir to the output layer units.

Unlike traditional neural networks, the matrices W_{in}, W , and W_{back} in an ESN are initialized randomly at the network's inception and typically do not require further training beyond the W_{out} matrix. The reservoir updates its state with each new input using the state update equation, and the output is determined through a state output equation. This setup minimizes the need for extensive training, making ESNs particularly efficient for handling complex time-series predictions. Reservoir computing frameworks like ESNs are known for their ability to become

[5] Li, Gang, et al. "Echo State Network with Bayesian Regularization for Forecasting Short-Term Power Production of Small Hydropower Plants." *MDPI*, Multidisciplinary Digital Publishing Institute, 27 Oct. 2015, www.mdpi.com/1996-1073/8/10/12228/htm.

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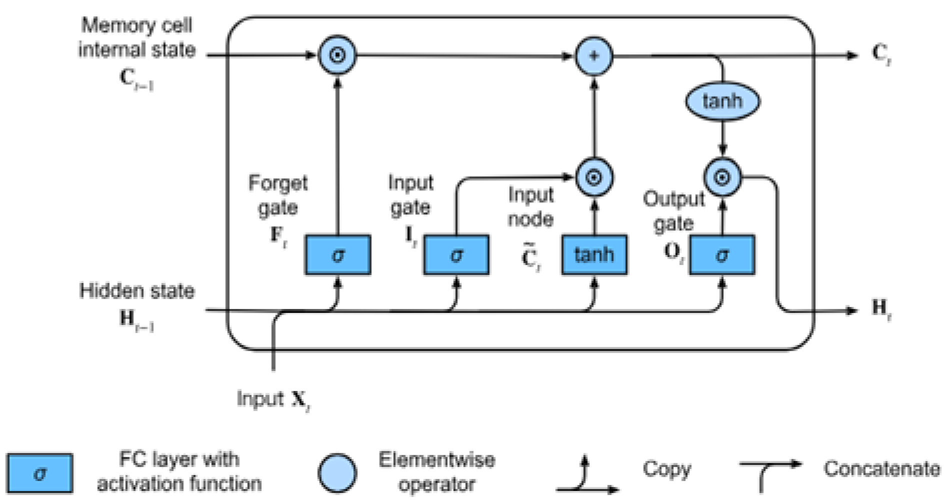
universal approximators for dynamical systems[6] such as the stock market - an intriguing quality that initially motivated this research.

Long Short Term Memory Networks (LSTM)

Long Short-Term Memory networks (LSTMs) are a special kind of Recurrent Neural Network (RNN) capable of learning long-term dependencies in data sequences. They were introduced by Sepp Hochreiter and Jürgen Schmidhuber in 1997 to address the vanishing gradient problem commonly associated with standard RNNs. As seen in Figure 3, the LSTM introduces gates that regulate the flow of information, including the input gate, forget gate, and output gate.

Figure 2: Barebones LSTM Architecture[7]



- Forget Gate (F_t): Decides what information to discard from the cell state. It looks at the previous hidden state (H_{t-1}) and the current input X_t, and applies a sigmoid function to determine the portions of the cell state to be removed.

- Input Gate (I_t): Decides what new information to store in the cell state. This gate includes a sigmoid layer that updates the cell state values and a tanh layer that creates a vector of new candidate values, ($\widehat{C}_t$), that could be added to the state.

[6] Li, Zhen, and Yunfei Yang. "Universality and Approximation Bounds for Echo State Networks with Random Weights." *arXiv.Org e-Print Archive*, 6 Dec. 2022, arxiv.org/pdf/2206.05669v1.
[7] "10.1. Long Short-Term Memory (LSTM" *10.1. Long Short-Term Memory (LSTM) - Dive into Deep Learning 1.0.3 Documentation*, d2l.ai/chapter_recurrent-modern/lstm.html. Accessed 12 May 2024.

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- Cell State (C_t): This is the "memory" part of the LSTM, modified by the forget gate and input gate. The old state (C_{t-1}) is multiplied by the forget gate's output, and then the input gate's output is added, which involves adding the candidate values scaled by how much new information we decided to include.

- Output Gate (O_t): Determines the next hidden state, which contains filtered information from the cell state used for predictions. The cell state is passed through a tanh function (to normalize values between -1 and 1), and then it is multiplied by the output of the sigmoid gate on the hidden state and input, deciding which parts of the cell state make it to the output.

This architecture allows them to effectively handle long-term dependencies and avoid the vanishing gradient problem that plagues standard RNNs. Consequently, LSTMs are particularly suited for tasks requiring memory of sequential data[8], such as time-series prediction, natural language processing, and complex decision-making processes. In this research paper, LSTMs are employed for their proficiency in modeling financial time series, where understanding long-term dependencies is crucial for accurate forecasting and risk assessment, making for a solid benchmark to compare ESNs to.

Gated Recurrent Units (GRU)

Gated Recurrent Units, commonly known as GRUs, were introduced by Cho et al. in 2014[9] as a streamlined alternative to the more intricate Long Short-Term Memory (LSTM) networks. Designed to capture long-range dependencies within sequence data, GRUs consolidate the functionality of the forget and input gates of an LSTM into one unified "update gate." as seen in Figure 3. Additionally, GRUs combine the cell state and hidden state into a singular state, thereby reducing computational demands compared to LSTMs while maintaining comparable efficacy. The inclusion of GRUs in this comparative analysis is primarily motivated by their simplified architecture, a feature that aligns well with the principles of common reservoir computing frameworks. While the simpler architecture does reduce training times, it may bring forth underfitting issues when dealing with complex systems like the stock market. The GRU model type was chosen for this research as its relative simplicity compared to LSTMs is reminiscent of the ESNs aforementioned architectural simplicity, where a comparison in this sense might provide additional insights.

[8] Shinde, Sagar et al. "Stock Price Prediction Using LSTM." *IEEE Xplore* , IEEE, 2023, ieeexplore.ieee.org/document/10392023.
[9] Cho, Kyunghyun, et al. "Learning phrase representations using RNN encoder-decoder for statistical machine translation." arXiv preprint arXiv:1406.1078 (2014)

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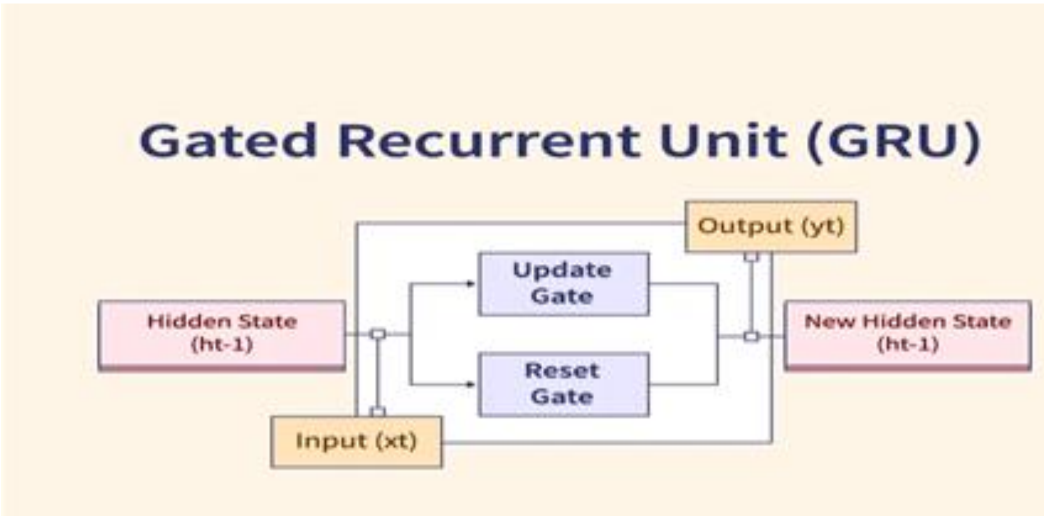
Figure 3: Barebones GRU Architecture[10]



Generalized Auto Regressive Conditional Heteroskedasticity (GARCH)

The Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model was introduced by Tim Bollerslev in 1986[11] who built upon the earlier work by Robert Engle, who introduced the Autoregressive Conditional Heteroskedasticity (ARCH) model in 1982[12]. Bollerslev's GARCH model extended the ARCH model to provide a more flexible and robust framework for modeling and forecasting time-varying volatility, which has since become a widely used benchmark in the analysis of financial time series - specifically, forecasting daily volatility. The GARCH(p, q) model can be represented with the following equations, where p p and q q denote the order of the GARCH and ARCH terms respectively:[13]

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i \epsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2$$

- σ_t^2 : Conditional variance of the current time period.

[10] Malingan, Navaneeth. "Gated Recurrent Unit (GRU)." *Scaler Topics*, Scaler Topics, 22 Feb. 2023, www.scaler.com/topics/deep-learning/gru-network/.
[11] Bollerslev, Tim, 1986. "Generalized Autoregressive Conditional Heteroskedasticity," Journal of Econometrics, Elsevier, vol. 31(3), pages 307-327.
[12] Engle, Robert, 1982. "Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of United Kingdom Inflation," Econometrica, Econometric Society, vol. 50(4), pages 987-1007.
[13] Zivot, Eric. "10.2 Bollerslev's GARCH Model." *BookDown*, 11 June 2021, bookdown.org/compfinezbook/introcompfinr/bollerslevs-garch-model.html.

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- ϵ_t^2 : Residual at time t, which is the difference between the actual return and the expected return.

- α_0 : Constant term.

- α_i : Coefficients of the ARCH terms (lagged squared residuals).

- β_j : Coefficients of the GARCH terms (lagged conditional variances).

While the ARCH term captures the impact of the previous time periods' forecast errors on the current volatility and thus reflects short-term effects, the GARCH term represents the persistence of volatility, or the impact of past conditional variances on the current period's variance, showing the long-term effects and is key to capturing the clustering of volatility. As stated prior, the GARCH model type is chosen for this research as a benchmark for comparing the performance of ESNs in forecasting daily volatility.

Methodology

Experimental Data

Nine datasets were used in this experiment, sourced from Yahoo! Finance's official website, comprising 1294 data points for each financial metric. This corresponds to five real life years, with the dates starting from October 15th, 2018, to December 6th, 2023.To test the performance of the models across varying market conditions, we strategically selected three types of financial metrics from each level of market volatility and analyzed them for additional insights into model performance.

Dataset Composition and Derivation of Financial Metrics:

The datasets are categorized into three distinct types, each representing different segments of the financial markets:

1. Market-Level Indices (Moderate to High Volatility):

- NASDAQ
- NYSE (New York Stock Exchange).
- SPX (S&P 500)

2. Exchange-Traded Funds (ETFs):

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- Invesco QQQ
- IShares
- Vanguard

3. Single Stocks (Varied Volatility):

- Amazon (AMZN)
- Google (Alphabet Inc.)
- Apple (AAPL)

By considering varying levels of volatility, additional insights can be made when analyzing model performance.

Calculation of Additional Financial Indicators:

To enhance our dataset, we manually derived two key financial metrics using formulas in Google Sheets:

Daily Volatility: Calculated using the formula $\frac{High-Low}{Close}$, this metric measures the intraday price movement relative to the closing price.

Log Returns: Determined by $ln(\frac{Close}{Open})$, this metric captures the relative price change from one day to the next in a format suitable for models that assume normally distributed returns.

The metrics used to compare the performance of ESNs against traditional RNNs were Close Price, Daily Volatility, and Log Returns. Closing prices are critical in financial markets as they represent the final agreed-upon price at the end of the trading day, reflecting the day's conclusive market sentiment. Log Returns simplify the aggregation of returns over time, making them especially useful in portfolio management and risk assessment. Daily Volatility provides a snapshot of the day's price stability or risk, offering insights into market uncertainty that are vital for risk management and trading strategies. Each of the chosen metrics capture different aspects of market dynamics—stability, trend, and risk—which are crucial for robust financial analysis and modeling.

Exploratory Data Analysis

This section provides a quick overview of the collected datasets, showing all columns, rows, and an exemplary heatmap for the SP500 dataset:

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Raw Dataset (Fig. 4):

Column Name	Description
Date	The specific trading day for each entry.
Open	The opening price of the stock at the beginning of the trading day.
High	The highest price at which the stock traded during the trading day.
Low	The lowest price at which the stock traded during the trading day.
Close	The closing price of the stock at the end of the trading day.
Adj Close	Adjusted closing price, modified for corporate actions like dividends, splits.
Volume	Total number of shares traded during the trading day.

Derived Dataset (Fig. 5)

Metric	Formula	Description
Log Returns	$ln(\frac{Close}{Open})$	Measures the change in price over the trading day.
Daily Volatility	$\frac{High - Low}{Close}$	Provides a relative measure of the intraday price range, indicative of the stock's stability or risk during the trading day.

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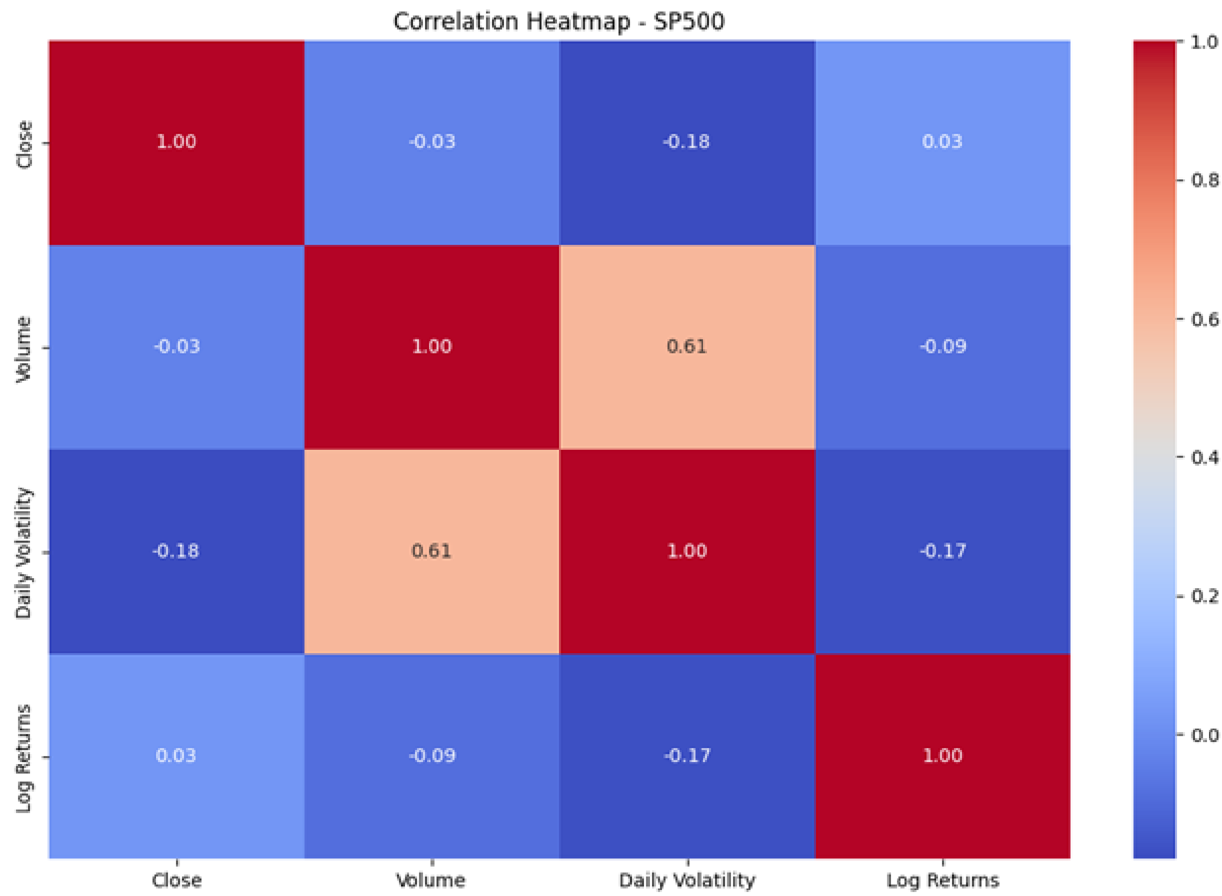
Correlation Heatmap (Fig. 6)



The correlation heatmap above provides a visual representation of the statistical correlations between different metrics in the dataset. Shades of blue indicate positive correlation, while shades of red indicate negative correlation, with intensity reflecting the strength of the relationship.

- Close and Volume: Shows a near-zero correlation suggesting that the day's trading volume does not necessarily affect the closing price significantly.
- Daily Volatility and Log Returns: Exhibits a positive correlation (0.61), indicating that days with higher volatility tend to have larger movements in log returns, reflecting higher risk and return potential.
- Volume and Log Returns: Shows a slightly negative correlation (-0.09), hinting that higher trading volumes might slightly coincide with lower price changes over the day.

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Data Preprocessing

Before experimentation, the datasets underwent specific data preprocessing steps to ensure quality and consistency. These steps are crucial for handling missing values, capping outliers, and correcting invalid data entries, which can significantly influence the outcomes of time series forecasting models.

1. Handling Missing Values:
   - Checked each dataset for the presence of null or missing values across all columns.
2. Capping Outliers in 'Daily Volatility':
   - Financial datasets sometimes exhibit extreme volatility, which can distort predictive modeling. To address this, identified values in the 'Daily Volatility' column exceeding an extreme value (100).
3. Correcting Non-Positive Values in Price Columns:
   - For the 'Open', 'High', 'Low', and 'Close' price columns, it is essential that all entries are positive, as negative or zero values are not feasible in this context. Screened these columns for non-positive values.

No issues related to the aforementioned checks were found within any of the datasets collected and derived from Yahoo Finance.

Evaluation Metrics

The assessment of the performance of various predictive models was conducted by evaluating the mean absolute error (MAE) and mean squared error (MSE) between predicted and observed values.

Mean Squared Error (MSE): MSE is a metric that evaluates the average of the squares of the errors, effectively assessing the variance between predicted values and observed data points. It is calculated by the formula:

$$MSE = \frac{1}{n}\sum_{i=1}^{n} (Y_i - \hat{Y}_i)^2$$

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Where (n) is the number of data points, (Y_i) represents the actual observed values, ($\hat{Y}_i$) represents the predicted values by the model. The squaring of the errors in MSE significantly penalizes larger discrepancies between the predicted and actual values.

Mean Absolute Error (MAE): MAE measures the average magnitude of errors in predictions, without considering the direction of these errors (i.e., whether they are positive or negative). It is defined as:xx

$$MAE = \frac{1}{n}\sum_{i=1}^{n} \left|Y_i - \hat{Y}_i\right|^2$$

Where, $|Y_i - \hat{Y}_i|^2$is the absolute error between the model's prediction and the actual value for each instance in the dataset. Unlike MSE, MAE is less influenced by outliers as it does not square the error values.

Hyperparameter Optimization

This section elaborates on the specific methods used to fine-tune each model and the rationale behind the selection of certain hyperparameters.

The common framework for our hyperparameter tuning involved Cross-Validation Grid Search (CVGS), a robust method for model optimization. CVGS evaluates a grid of hyperparameter combinations and selects the combination that performs best, according to a predefined metric, in our case Mean Squared Error (MSE) . This process helps in identifying the most effective model settings that minimize forecasting errors on unseen data.

GARCH Model Tuning

For the GARCH model, primarily used for time series analysis of financial data where volatility clustering is evident, we tuned the lag order parameters q and p:

- *p:* Number of lag observations included in the model (autoregressive terms).
- *q:* Number of lag forecast errors in the prediction equation (moving average terms).

The tuning involved iterating over a range of values for q and p from 1 to 3, fitting the GARCH model to the data, and selecting the parameters that resulted in the lowest Akaike Information Criterion (AIC), indicative of the best fit with minimal information loss.

ESN Model Tuning

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For the ESN models, tuning involved the following hyperparameters:

- *Reservoir size (number of neurons):* Influences the capacity and memory of the network.
- *Spectral radius:* Affects the stability and dynamics of the network state.
- *Sparsity:* Determines the percentage of zero-weight connections in the reservoir.
- *Noise:* Adds stochasticity to the state updates, promoting robustness.

Each combination's performance was assessed using MSE.

GRU and LSTM Model Tuning

For the GRU and LSTM models, which are types of recurrent neural networks effective in capturing temporal dependencies in sequence data, we optimized:

- *Number of neurons (units in LSTM or GRU layers):* Controls the model's complexity and ability to learn from data.
- *Dropout rate:* Regularization parameter to prevent overfitting by randomly dropping units during training.
- *Input sequence length (window size for input features):* Impacts how much past information the model considers for forecasting.

Similar to ESN, each combination's performance was assessed using MSE.

For each model, the tuning process was repeated for the target variables 'Close', 'Daily Volatility', and 'Log Returns', considering different prediction horizons (1, 5, 18, 30, 72 days into future).

Experimental Settings

Prior to the training phase, the dataset was divided into two separate segments: an 80% training set and a 20% testing set. The larger portion, the training set, was employed to train the models on the intrinsic patterns present within the financial data. The smaller segment, the testing set, was reserved solely for assessing the effectiveness of the models after training. The model hyperparameters for train/test phase were obtained from the aforementioned Cross Validation Grid Search algorithm.

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The operating environment was Python 3.10 and Google Colaboratory (a hosted Jupyter notebook service) on Windows 10 (64 bit). We used the pandas library to read the datasets and conduct experiments, the arch library to build GARCH models, the tensorflow.keras library to build LSTM and GRU models, and the ESN class code was imported from the Pytorch-ESN module[14].

Results

Given the five prediction horizons - (1,5,18,30,72 days into future) - for three financial metrics - Close Price, Daily Volatility, Log Returns - the number of result tables amounted to fifteen. Given the sheer number of data tables, a binary best/worst system color coded by "green" and "red" was used to compare model performances for the prediction tasks. For each task, the model with the lowest MSE and MAE was coded "green", while the model with the highest MSE and MAE was coded "red". The system is exemplified as follows (all other tables are found in the Appendix):

Table 4 (Close Price Prediction, 30th day into future)

	Echo State Network		GARCH		LSTM		GRU	
Dataset	MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE
Amazon	4.19	2.05	254	15.0	6.43	2.05	8.24	2.30
Google	43.7	6.61	914	29.9	9.54	2.54	42.4	6.14
Apple	139	11.8	1400	36.5	32.2	4.72	409	19.5
NASDAQ	7380000	2720	4590000	2070	191000	365	252000	417
SP500	393	19.8	105000	289	24600	136	28000	144
NYSE	13100	115	239000	417	90800	233	162000	351
ISHARES	12.6	3.55	9490	95.4	37.0	4.99	58.8	6.79

[14] Onardo, Stefan. "Pytorch-Esn/Torchesn/Nn/Echo_state_network.Py at Master · Stefanonardo/Pytorch-ESN." *GitHub*, 2018, github.com/stefanonardo/pytorch-esn/blob/master/torchesn/nn/echo_state_network.py.

QQQ	21.6	4.65	4740	67.1	115	9.14	60.0	6.31
VANGUARD	0.533	0.730	9.77	2.38	2.45	1.22	13.5	3.22

Table 7 (Daily Volatility Prediction, 5th day into future)

	Echo State Network		GARCH		LSTM		GRU	
Dataset	MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE
Amazon	$1.87 \cdot 10^{-4}$	$1.37 \cdot 10^{-2}$	$7.62 \cdot 10^{-5}$	$7.4 \cdot 10^{-3}$	$3.33 \cdot 10^{-5}$	$4.61 \cdot 10^{-3}$	$1.00 \cdot 10^{-4}$	$8.05 \cdot 10^{-3}$
Google	$7.86 \cdot 10^{-6}$	$2.80 \cdot 10^{-3}$	$6.80 \cdot 10^{-5}$	$7.82 \cdot 10^{-3}$	$4.46 \cdot 10^{-4}$	$1.07 \cdot 10^{-2}$	$3.62 \cdot 10^{-5}$	$4.04 \cdot 10^{-3}$
Apple	$7.28 \cdot 10^{-6}$	$2.70 \cdot 10^{-3}$	$4.56 \cdot 10^{-5}$	$6.27 \cdot 10^{-3}$	$2.51 \cdot 10^{-4}$	$1.24 \cdot 10^{-2}$	$5.16 \cdot 10^{-5}$	$5.84 \cdot 10^{-3}$
NASDAQ	$2.27 \cdot 10^{-5}$	$4.47 \cdot 10^{-3}$	$4.00 \cdot 10^{-6}$	$1.64 \cdot 10^{-3}$	$6.05 \cdot 10^{-5}$	$7.52 \cdot 10^{-3}$	$2.08 \cdot 10^{-5}$	$3.50 \cdot 10^{-3}$
SP500	$1.48 \cdot 10^{-6}$	$1.21 \cdot 10^{-3}$	$1.14 \cdot 10^{-5}$	$2.85 \cdot 10^{-3}$	$1.99 \cdot 10^{-5}$	$4.12 \cdot 10^{-3}$	$1.97 \cdot 10^{-5}$	$3.87 \cdot 10^{-3}$
NYSE	$4.49 \cdot 10^{-6}$	$2.23 \cdot 10^{-3}$	$5.91 \cdot 10^{-6}$	$2.23 \cdot 10^{-3}$	$3.17 \cdot 10^{-5}$	$4.09 \cdot 10^{-3}$	$1.01 \cdot 10^{-5}$	$2.81 \cdot 10^{-3}$
ISHARES	$8.41 \cdot 10^{-5}$	$9.17 \cdot 10^{-3}$	$7.15 \cdot 10^{-6}$	$2.45 \cdot 10^{-3}$	$4.79 \cdot 10^{-5}$	$5.84 \cdot 10^{-3}$	$3.92 \cdot 10^{-5}$	$4.87 \cdot 10^{-3}$
QQQ	$3.47 \cdot 10^{-5}$	$5.89 \cdot 10^{-3}$	$1.01 \cdot 10^{-5}$	$2.79 \cdot 10^{-3}$	$4.03 \cdot 10^{-5}$	$6.16 \cdot 10^{-3}$	$9.42 \cdot 10^{-5}$	$9.10 \cdot 10^{-3}$
VANGUARD	$4.13 \cdot 10^{-5}$	$6.42 \cdot 10^{-3}$	$7.10 \cdot 10^{-6}$	$2.41 \cdot 10^{-3}$	$5.85 \cdot 10^{-5}$	$5.55 \cdot 10^{-3}$	$1.90 \cdot 10^{-5}$	$3.82 \cdot 10^{-3}$

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Table 14 (Log Returns Prediction, 30th day into future)

	Echo State Network		GARCH		LSTM		GRU	
Dataset	MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE
Amazon	$2.72 \cdot 10^{-5}$	$5.21 \cdot 10^{-3}$	$4.59 \cdot 10^{-4}$	$1.53 \cdot 10^{-2}$	$4.83 \cdot 10^{-4}$	$1.88 \cdot 10^{-2}$	$4.83 \cdot 10^{-4}$	$1.87 \cdot 10^{-2}$
Google	$8.44 \cdot 10^{-4}$	$2.91 \cdot 10^{-2}$	$4.96 \cdot 10^{-4}$	$1.38 \cdot 10^{-2}$	$3.91 \cdot 10^{-4}$	$1.47 \cdot 10^{-2}$	$3.90 \cdot 10^{-4}$	$1.49 \cdot 10^{-2}$
Apple	$2.18 \cdot 10^{-4}$	$1.48 \cdot 10^{-2}$	$1.23 \cdot 10^{-4}$	$8.93 \cdot 10^{-3}$	$4.51 \cdot 10^{-4}$	$1.69 \cdot 10^{-2}$	$4.52 \cdot 10^{-4}$	$1.69 \cdot 10^{-2}$
NASDAQ	$2.61 \cdot 10^{-5}$	$5.11 \cdot 10^{-3}$	$1.08 \cdot 10^{-4}$	$7.58 \cdot 10^{-3}$	$2.51 \cdot 10^{-4}$	$1.25 \cdot 10^{-2}$	$2.51 \cdot 10^{-4}$	$1.23 \cdot 10^{-2}$
SP500	$6.18 \cdot 10^{-7}$	$7.86 \cdot 10^{-4}$	$6.21 \cdot 10^{-5}$	$5.74 \cdot 10^{-3}$	$1.53 \cdot 10^{-4}$	$9.43 \cdot 10^{-3}$	$1.54 \cdot 10^{-4}$	$9.45 \cdot 10^{-3}$
NYSE	$2.22 \cdot 10^{-6}$	$1.49 \cdot 10^{-3}$	$6.21 \cdot 10^{-5}$	$6.08 \cdot 10^{-3}$	$1.14 \cdot 10^{-4}$	$8.68 \cdot 10^{-3}$	$1.13 \cdot 10^{-4}$	$8.65 \cdot 10^{-3}$
ISHARES	$9.60 \cdot 10^{-5}$	$9.80 \cdot 10^{-3}$	$1.42 \cdot 10^{-4}$	$8.62 \cdot 10^{-3}$	$3.35 \cdot 10^{-4}$	$1.45 \cdot 10^{-2}$	$3.41 \cdot 10^{-4}$	$1.43 \cdot 10^{-2}$
QQQ	$1.04 \cdot 10^{-5}$	$3.23 \cdot 10^{-3}$	$1.09 \cdot 10^{-4}$	$7.60 \cdot 10^{-3}$	$2.60 \cdot 10^{-4}$	$1.25 \cdot 10^{-2}$	$2.60 \cdot 10^{-4}$	$1.25 \cdot 10^{-2}$
VANGUARD	$8.39 \cdot 10^{-5}$	$9.16 \cdot 10^{-3}$	$6.20 \cdot 10^{-5}$	$6.19 \cdot 10^{-3}$	$9.72 \cdot 10^{-5}$	$8.02 \cdot 10^{-3}$	$9.76 \cdot 10^{-5}$	$7.89 \cdot 10^{-3}$

By quantifying the relative success/failure of the different models, the following result analysis was possible:

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Table 16 : Overview of Best Performing Models										
	1st day		5th day		18th day		30th day		72nd day	
Metric	**Best model**	**B/W Ratio**	**Best model**	**B/W Ratio**	**Best model**	**B/W Ratio**	**Best model**	**B/W Ratio**	**Best model**	**B/W Ratio**
Close Price	LSTM	7/0	ESN	4/0	ESN	3/0	ESN	6/1	ESN	5/2
Daily Volatility	GARCH	5/0	GARCH	4/0	GARCH	6/0	GARCH	7/0	GARCH	8/0
Log Returns	LSTM	4/0	ESN	6/1	GARCH	5/0	ESN	6/1	GARCH	5/0

**B/W Ratio = n times model was best (green)/n times model was worst (red)*

For a model to be qualified as "best", they had to have the highest value of number of bests - number of worsts for a given task. Table 16 shows a high-level overview of the best performing models for each task. Two striking observations are that the Echo State Network (ESN) model outperforms its peers consistently for 80% of the Close Price forecasting tasks while being overshadowed by the GARCH model in Daily Volatility prediction tasks. The ESN model shows particularly strong results for close price predictions over longer periods (30th and 72nd day) and for log returns on the 72nd day, indicating a strong capability in capturing longer-term dependencies in the data. In the context of predicting close prices, the model's high performance despite longer prediction horizons could be attributed to its ability to forecast the intrinsic value of the stock/ETF/index rather than merely reacting to market sentiments. Despite being overshadowed by GARCH in daily volatility forecasting, the ESN model outperformed LSTM and GRU models consistently.

Table 17 : Models & General Performance		
Model	**Number of Bests**	**Number of Worsts**
ESN	52*	32
GARCH	50	35
LSTM	22	27
GRU	11	35

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Table 17 shows the cumulative number of bests vs. number of worsts for each model type. The ESN model has the highest total number of bests at 52, followed closely by the GARCH model at 50, and a steep drop off to 22 and 11 with LSTM and GRU models. It's important to note that while the GARCH model, based on this table, looks to be generally performing close to the ESN model - a large majority of these "bests" are from daily volatility forecasting tasks (30/50). Thus, it appears that the ESN outperforms its peers in financial metric forecasting. This is especially interesting, as the architecturally simplest model, the ESN, performs better than more complex models like LSTM and GRU, highlighting its aforementioned ability to become universal approximators for dynamic systems like the stock market.

The proficient forecasting capabilities of Echo State Networks (ESNs) in predicting financial metrics can be transformative for various players within the financial sector. Investment analysts and fund managers can significantly benefit from the ESN's accuracy in predicting long-term price movements, utilizing these insights to refine investment strategies and enhance portfolio returns. Risk managers may leverage the ESN's ability to forecast log returns and close prices over different periods to enhance risk assessment and management strategies, particularly in volatile environments. For retail investors, the ESN offers a tool to base trading decisions on the intrinsic value of assets rather than market sentiment, potentially leading to more informed and successful investment choices.

Future Work

The promising results of Echo State Networks (ESN) in forecasting daily financial metrics raise several questions and opportunities for further research. Future investigations could explore the versatility and robustness of ESNs across different applications and conditions:

1. High-Frequency Data Analysis: It remains to be seen whether ESNs maintain their performance advantage in environments characterized by high-frequency trading data, such as minute-by-minute or hourly price updates. Analyzing ESNs in such settings could provide insights into their adaptability and effectiveness in capturing rapid market dynamics.

2. Diverse Asset Classes: Extending the application of ESNs beyond stocks and indices to include a broader range of asset classes such as bonds, cryptocurrencies, real estate, alternative assets, and commodities (i.e. gold and silver) would help in understanding the model's effectiveness across different investment domains. This would also allow for a comprehensive analysis of how ESNs handle various asset behaviors and risk profiles.

3. Portfolio Application and Correlation Analysis: Investigating the implications of using ESNs in portfolio management, particularly how they predict the correlation and log

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returns of different assets under varying market volatility levels, could be highly beneficial. This research could lead to better risk management strategies and enhanced decision-making processes in portfolio optimization, especially in complex and fluctuating financial environments.

Conclusions

Financial metric forecasting has time-series features. ESNs are a newer type of RNN that utilize reservoir computing features. LSTM, GRU, and GARCH are RNNs widely used for predicting financial metrics. The GARCH model, while exceptional in modeling daily volatility, often did not perform as well in other financial metrics, highlighting the specialized nature of GARCH in handling volatility clustering. Conversely, LSTMs and GRUs, despite their architectural sophistication, did not consistently deliver superior performance - suggesting that a simplified LSTM/GRU architecture might perform better. The ESN models had the highest cumulative number of "bests", and showed exceptional performance in close price forecasting. Further, the accuracy of ESNs, measured by MSE and MAE, didn't drop off substantially when approaching longer prediction horizons, indicating a strong capability in capturing longer-term dependencies. Overall, the ESN outperformed its peers in financial metric forecasting. The ability of ESNs to approximate dynamic systems effectively makes them particularly suitable for financial markets, where predicting future values involves understanding both the underlying patterns and potential irregularities within vast datasets. In future work, we aim to narrow the research's focus on one metric, and analyze a higher number of prediction horizons to paint a more accurate picture of model performance for a specific task.

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Appendix

Table 1 (Close Price Prediction, 1st day into future)

	Echo State Network		GARCH		LSTM		GRU	
Dataset	MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE
Amazon	16.0	4.00	313.4	17.7	0.05	0.224	7.78	2.79
Google	4.80	2.19	810.3	28.5	$3.32 \cdot 10^{-3}$	$5.77 \cdot 10^{-2}$	7.70	2.77
Apple	37.8	6.15	2018.1	44.9	5.70	2.39	4.39	2.10
NASDAQ	$4.52 \cdot 10^6$	$2.13 \cdot 10^3$	$6.03 \cdot 10^6$	2457.2	$1.66 \cdot 10^4$	$1.29 \cdot 10^2$	$1.66 \cdot 10^5$	$4.08 \cdot 10^2$
SP500	13042.3	114.2	$1.70 \cdot 10^5$	411.9	3730.0	61.1	739.0	27.2
NYSE	$4.23 \cdot 10^4$	$2.06 \cdot 10^2$	$4.78 \cdot 10^5$	691.2	3390.0	58.3	$7.62 \cdot 10^4$	$2.76 \cdot 10^2$
ISHARES	217	14.7	$1.13 \cdot 10^4$	106.4	4.38	2.09	266.0	16.3

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QQQ	138	11.7	5814.6	76.3	$1.44 \cdot 10^{-3}$	$3.79 \cdot 10^{-2}$	32.3	5.69
VANGUARD	11.6	3.40	1.33	1.15	0.234	0.484	1.04	1.02

Table 2 (Close Price Prediction, 5th day into future)								
	Echo State Network		GARCH		LSTM		GRU	
Dataset	MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE
Amazon	41.4	6.43	364.1	19.1	2.73	1.36	3.36	1.58
Google	15.4	3.93	864.2	29.4	49.7	5.57	30.2	5.31
Apple	5.06	2.25	1927.1	43.9	7.89	2.47	6.57	2.09
NASDAQ	$9.68 \cdot 10^{5}$	$9.84 \cdot 10^{2}$	$6.40 \cdot 10^{6}$	2528.2	$1.20 \cdot 10^{5}$	$3.14 \cdot 10^{2}$	$3.35 \cdot 10^{5}$	$5.62 \cdot 10^{2}$
SP500	1420.3	37.7	$1.87 \cdot 10^{5}$	432.3	3420.0	58.2	5260.0	56.1
NYSE	$2.20 \cdot 10^{5}$	$4.69 \cdot 10^{2}$	$5.81 \cdot 10^{5}$	759.0	$1.66 \cdot 10^{5}$	$3.68 \cdot 10^{2}$	$1.54 \cdot 10^{5}$	$3.82 \cdot 10^{2}$
ISHARES	83.6	9.14	$1.23 \cdot 10^{4}$	110.5	219.0	12.1	577.0	23.5
QQQ	41.9	6.47	6195.0	78.7	$3.26 \cdot 10^{2}$	14.3	15.4	3.34
VANGUARD	3.20	1.79	2.50	1.51	16.1	3.62	30.3	4.80

Table 3 (Close Price Prediction, 18th day into future)

	Echo State Network		GARCH		LSTM		GRU	
Dataset	MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE
Amazon	93.0	9.64	347.6	18.6	83.8	7.82	5.62	1.90
Google	59.5	7.71	1086.7	32.3	99.3	9.28	67.5	7.80
Apple	321	17.9	1797.0	42.3	157	10.0	7.76	2.13
NASDAQ	$4.90 \cdot 10^5$	$7.00 \cdot 10^2$	$6.12 \cdot 10^6$	2469.4	$9.02 \cdot 10^4$	$2.52 \cdot 10^2$	$3.21 \cdot 10^6$	$1.57 \cdot 10^3$
SP500	45225.2	212.7	$1.57 \cdot 10^5$	393.3	$4.79 \cdot 10^4$	194.8	$2.32 \cdot 10^4$	135.0
NYSE	$2.83 \cdot 10^5$	$5.32 \cdot 10^2$	$2.96 \cdot 10^5$	493.5	$6.64 \cdot 10^5$	$7.40 \cdot 10^2$	$1.51 \cdot 10^6$	$1.08 \cdot 10^3$
ISHARES	334	18.3	$1.21 \cdot 10^4$	109.8	650.0	23.1	4810.0	61.7
QQQ	394	19.8	6111.4	78.1	$1.34 \cdot 10^3$	33.1	23.0	3.98
VANGUARD	27.0	5.20	2.12	1.08	75.5	7.89	147.0	11.1

Table 4 (Close Price Prediction, 30th day into future)

	Echo State Network		GARCH		LSTM		GRU	
Dataset	MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE
Amazon	4.19	2.05	254	15.0	6.43	2.05	8.24	2.30
Google	43.7	6.61	914	29.9	9.54	2.54	42.4	6.14

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Apple	139	11.8	1400	36.5	32.2	4.72	409	19.5
NASDAQ	7380000	2720	4590000	2070	191000	365	252000	417
SP500	393	19.8	105000	289	24600	136	28000	144
NYSE	13100	115	239000	417	90800	233	162000	351
ISHARES	12.6	3.55	9490	95.4	37.0	4.99	58.8	6.79
QQQ	21.6	4.65	4740	67.1	115	9.14	60.0	6.31
VANGUARD	0.533	0.730	9.77	2.38	2.45	1.22	13.5	3.22

Table 5 (Close Price Prediction, 72nd day into future)

	Echo State Network		GARCH		LSTM		GRU	
Dataset	MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE
Amazon	2.76	1.66	144	9.87	93.3	7.66	239	12.4
Google	40.2	6.34	1040	32.0	106	8.24	177	11.8
Apple	4.24	2.06	1120	32.8	523	18.4	2020	40.4
NASDAQ	1760000	1330	3920000	1930	274000	440	285000	453
SP500	63800	253	80900	258	14000	97.2	16900	106
NYSE	1780000	1340	191000	368	1300000	943	113000	280

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ISHARES	52.0	7.21	7910	87.5	1220	27.4	909	24.2
QQQ	1.69	1.30	3940	61.5	280	14.1	1970	34.9
VANGUARD	63.0	7.94	8.59	2.40	25.1	4.16	15.3	3.48

Table 6 (Daily Volatility Prediction, 1st day into future)

	Echo State Network		GARCH		LSTM		GRU	
Dataset	MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE
Amazon	$6.06 \cdot 10^{-4}$	$2.46 \cdot 10^{-2}$	$5.78 \cdot 10^{-6}$	$2.40 \cdot 10^{-3}$	$3.18 \cdot 10^{-5}$	$5.64 \cdot 10^{-3}$	$2.85 \cdot 10^{-6}$	$1.69 \cdot 10^{-3}$
Google	$4.18 \cdot 10^{-5}$	$6.94 \cdot 10^{-3}$	$2.51 \cdot 10^{-5}$	$5.01 \cdot 10^{-3}$	$6.94 \cdot 10^{-6}$	$2.63 \cdot 10^{-3}$	$1.50 \cdot 10^{-4}$	$1.23 \cdot 10^{-2}$
Apple	$2.96 \cdot 10^{-5}$	$5.44 \cdot 10^{-3}$	$4.06 \cdot 10^{-5}$	$6.37 \cdot 10^{-3}$	$1.57 \cdot 10^{-5}$	$3.97 \cdot 10^{-3}$	$1.18 \cdot 10^{-4}$	$1.09 \cdot 10^{-2}$
NASDAQ	$7.57 \cdot 10^{-6}$	$2.75 \cdot 10^{-3}$	$4.26 \cdot 10^{-9}$	$6.53 \cdot 10^{-5}$	$2.27 \cdot 10^{-5}$	$4.76 \cdot 10^{-3}$	$8.04 \cdot 10^{-5}$	$8.97 \cdot 10^{-3}$
SP500	$2.00 \cdot 10^{-5}$	$4.47 \cdot 10^{-3}$	$5.57 \cdot 10^{-7}$	$7.47 \cdot 10^{-4}$	$1.09 \cdot 10^{-5}$	$3.30 \cdot 10^{-3}$	$3.46 \cdot 10^{-5}$	$5.88 \cdot 10^{-3}$
NYSE	$8.21 \cdot 10^{-6}$	$2.86 \cdot 10^{-3}$	$2.75 \cdot 10^{-7}$	$5.25 \cdot 10^{-4}$	$9.34 \cdot 10^{-7}$	$9.66 \cdot 10^{-4}$	$9.04 \cdot 10^{-6}$	$2.83 \cdot 10^{-3}$
ISHARES	$9.69 \cdot 10^{-6}$	$3.11 \cdot 10^{-3}$	$1.01 \cdot 10^{-5}$	$3.17 \cdot 10^{-3}$	$6.00 \cdot 10^{-5}$	$7.75 \cdot 10^{-3}$	$7.05 \cdot 10^{-5}$	$8.40 \cdot 10^{-3}$
QQQ	$1.81 \cdot 10^{-5}$	$4.25 \cdot 10^{-3}$	$1.25 \cdot 10^{-9}$	$3.54 \cdot 10^{-5}$	$2.74 \cdot 10^{-5}$	$5.24 \cdot 10^{-3}$	$1.02 \cdot 10^{-5}$	$3.19 \cdot 10^{-3}$
VANGUARD	$5.14 \cdot 10^{-5}$	$7.17 \cdot 10^{-3}$	$3.35 \cdot 10^{-6}$	$1.83 \cdot 10^{-3}$	$1.35 \cdot 10^{-5}$	$3.67 \cdot 10^{-3}$	$1.99 \cdot 10^{-5}$	$4.46 \cdot 10^{-3}$

Table 7 (Daily Volatility Prediction, 5th day into future)

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	Echo State Network		GARCH		LSTM		GRU	
Dataset	MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE
Amazon	$1.87 \cdot 10^{-4}$	$1.37 \cdot 10^{-2}$	$7.62 \cdot 10^{-5}$	$7.4 \cdot 10^{-3}$	$3.33 \cdot 10^{-5}$	$4.61 \cdot 10^{-3}$	$1.00 \cdot 10^{-4}$	$8.05 \cdot 10^{-3}$
Google	$7.86 \cdot 10^{-6}$	$2.80 \cdot 10^{-3}$	$6.80 \cdot 10^{-5}$	$7.82 \cdot 10^{-3}$	$4.46 \cdot 10^{-4}$	$1.07 \cdot 10^{-2}$	$3.62 \cdot 10^{-5}$	$4.04 \cdot 10^{-3}$
Apple	$7.28 \cdot 10^{-6}$	$2.70 \cdot 10^{-3}$	$4.56 \cdot 10^{-5}$	$6.27 \cdot 10^{-3}$	$2.51 \cdot 10^{-4}$	$1.24 \cdot 10^{-2}$	$5.16 \cdot 10^{-5}$	$5.84 \cdot 10^{-3}$
NASDAQ	$2.27 \cdot 10^{-5}$	$4.47 \cdot 10^{-3}$	$4.00 \cdot 10^{-6}$	$1.64 \cdot 10^{-3}$	$6.05 \cdot 10^{-5}$	$7.52 \cdot 10^{-3}$	$2.08 \cdot 10^{-5}$	$3.50 \cdot 10^{-3}$
SP500	$1.48 \cdot 10^{-6}$	$1.21 \cdot 10^{-3}$	$1.14 \cdot 10^{-5}$	$2.85 \cdot 10^{-3}$	$1.99 \cdot 10^{-5}$	$4.12 \cdot 10^{-3}$	$1.97 \cdot 10^{-5}$	$3.87 \cdot 10^{-3}$
NYSE	$4.49 \cdot 10^{-6}$	$2.23 \cdot 10^{-3}$	$5.91 \cdot 10^{-6}$	$2.23 \cdot 10^{-3}$	$3.17 \cdot 10^{-5}$	$4.09 \cdot 10^{-3}$	$1.01 \cdot 10^{-5}$	$2.81 \cdot 10^{-3}$
ISHARES	$8.41 \cdot 10^{-5}$	$9.17 \cdot 10^{-3}$	$7.15 \cdot 10^{-6}$	$2.45 \cdot 10^{-3}$	$4.79 \cdot 10^{-5}$	$5.84 \cdot 10^{-3}$	$3.92 \cdot 10^{-5}$	$4.87 \cdot 10^{-3}$
QQQ	$3.47 \cdot 10^{-5}$	$5.89 \cdot 10^{-3}$	$1.01 \cdot 10^{-5}$	$2.79 \cdot 10^{-3}$	$4.03 \cdot 10^{-5}$	$6.16 \cdot 10^{-3}$	$9.42 \cdot 10^{-5}$	$9.10 \cdot 10^{-3}$
VANGUARD	$4.13 \cdot 10^{-5}$	$6.42 \cdot 10^{-3}$	$7.10 \cdot 10^{-6}$	$2.41 \cdot 10^{-3}$	$5.85 \cdot 10^{-5}$	$5.55 \cdot 10^{-3}$	$1.90 \cdot 10^{-5}$	$3.82 \cdot 10^{-3}$

Table 8 (Daily Volatility Prediction, 18th day into future)

	Echo State Network		GARCH		LSTM		GRU	
Dataset	MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE
Amazon	$3.16 \cdot 10^{-4}$	$1.78 \cdot 10^{-2}$	$5.03 \cdot 10^{-5}$	$6.20 \cdot 10^{-3}$	$9.84 \cdot 10^{-5}$	$6.78 \cdot 10^{-3}$	$4.41 \cdot 10^{-4}$	$2.33 \cdot 10^{-2}$
Google	$6.45 \cdot 10^{-7}$	$8.03 \cdot 10^{-4}$	$4.82 \cdot 10^{-5}$	$6.09 \cdot 10^{-3}$	$2.08 \cdot 10^{-4}$	$8.35 \cdot 10^{-3}$	$1.16 \cdot 10^{-4}$	$8.22 \cdot 10^{-3}$

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Apple	$1.83 \cdot 10^{-4}$	$1.35 \cdot 10^{-2}$	$8.00 \cdot 10^{-5}$	$8.47 \cdot 10^{-3}$	$1.31 \cdot 10^{-4}$	$8.90 \cdot 10^{-3}$	$9.98 \cdot 10^{-5}$	$7.76 \cdot 10^{-3}$
NASDAQ	$2.00 \cdot 10^{-4}$	$1.42 \cdot 10^{-2}$	$2.30 \cdot 10^{-5}$	$4.09 \cdot 10^{-3}$	$5.57 \cdot 10^{-5}$	$6.64 \cdot 10^{-3}$	$1.71 \cdot 10^{-4}$	$1.13 \cdot 10^{-2}$
SP500	$5.11 \cdot 10^{-5}$	$1.21 \cdot 10^{-3}$	$2.25 \cdot 10^{-5}$	$4.08 \cdot 10^{-3}$	$3.31 \cdot 10^{-5}$	$4.45 \cdot 10^{-3}$	$3.32 \cdot 10^{-5}$	$4.45 \cdot 10^{-3}$
NYSE	$1.90 \cdot 10^{-5}$	$4.35 \cdot 10^{-3}$	$2.38 \cdot 10^{-5}$	$4.02 \cdot 10^{-3}$	$1.02 \cdot 10^{-4}$	$8.37 \cdot 10^{-3}$	$8.81 \cdot 10^{-5}$	$7.91 \cdot 10^{-3}$
ISHARES	$1.58 \cdot 10^{-4}$	$1.26 \cdot 10^{-2}$	$3.50 \cdot 10^{-5}$	$5.22 \cdot 10^{-3}$	$4.91 \cdot 10^{-5}$	$5.44 \cdot 10^{-3}$	$4.82 \cdot 10^{-5}$	$5.03 \cdot 10^{-3}$
QQQ	$2.00 \cdot 10^{-4}$	$1.41 \cdot 10^{-2}$	$3.56 \cdot 10^{-5}$	$5.24 \cdot 10^{-3}$	$5.22 \cdot 10^{-5}$	$6.56 \cdot 10^{-3}$	$1.67 \cdot 10^{-4}$	$1.15 \cdot 10^{-2}$
VANGUARD	$1.23 \cdot 10^{-5}$	$3.51 \cdot 10^{-3}$	$1.50 \cdot 10^{-5}$	$3.45 \cdot 10^{-3}$	$1.29 \cdot 10^{-4}$	$9.60 \cdot 10^{-3}$	$2.06 \cdot 10^{-5}$	$3.59 \cdot 10^{-3}$

Table 9 (Daily Volatility Prediction, 30th day into future)

	Echo State Network		**GARCH**		**LSTM**		**GRU**	
Dataset	MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE
Amazon	$1.37 \cdot 10^{-4}$	$1.17 \cdot 10^{-2}$	$6.88 \cdot 10^{-5}$	$6.95 \cdot 10^{-3}$	$1.52 \cdot 10^{-4}$	$8.76 \cdot 10^{-3}$	$1.25 \cdot 10^{-4}$	$8.90 \cdot 10^{-3}$
Google	$1.56 \cdot 10^{-5}$	$3.96 \cdot 10^{-3}$	$4.32 \cdot 10^{-5}$	$5.64 \cdot 10^{-3}$	$1.10 \cdot 10^{-4}$	$6.59 \cdot 10^{-3}$	$1.12 \cdot 10^{-4}$	$8.16 \cdot 10^{-3}$
Apple	$9.62 \cdot 10^{-5}$	$9.81 \cdot 10^{-3}$	$6.68 \cdot 10^{-5}$	$7.37 \cdot 10^{-3}$	$1.12 \cdot 10^{-4}$	$8.42 \cdot 10^{-3}$	$1.10 \cdot 10^{-4}$	$8.72 \cdot 10^{-3}$
NASDAQ	$1.00 \cdot 10^{-4}$	$1.00 \cdot 10^{-2}$	$1.96 \cdot 10^{-5}$	$3.61 \cdot 10^{-3}$	$6.82 \cdot 10^{-5}$	$6.17 \cdot 10^{-3}$	$6.61 \cdot 10^{-5}$	$6.65 \cdot 10^{-3}$
SP500	$5.91 \cdot 10^{-5}$	$7.69 \cdot 10^{-3}$	$1.58 \cdot 10^{-5}$	$3.24 \cdot 10^{-3}$	$4.52 \cdot 10^{-5}$	$5.69 \cdot 10^{-3}$	$3.95 \cdot 10^{-5}$	$4.91 \cdot 10^{-3}$
NYSE	$2.09 \cdot 10^{-5}$	$4.57 \cdot 10^{-3}$	$2.19 \cdot 10^{-5}$	$3.74 \cdot 10^{-3}$	$3.03 \cdot 10^{-5}$	$4.59 \cdot 10^{-3}$	$3.37 \cdot 10^{-5}$	$4.55 \cdot 10^{-3}$

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ISHARES	$1.63 \cdot 10^{-4}$	$1.28 \cdot 10^{-2}$	$3.71 \cdot 10^{-5}$	$5.28 \cdot 10^{-3}$	$7.71 \cdot 10^{-5}$	$6.11 \cdot 10^{-3}$	$1.00 \cdot 10^{-4}$	$6.48 \cdot 10^{-3}$
QQQ	$1.10 \cdot 10^{-4}$	$1.05 \cdot 10^{-2}$	$2.86 \cdot 10^{-5}$	$4.64 \cdot 10^{-3}$	$8.50 \cdot 10^{-5}$	$6.16 \cdot 10^{-3}$	$7.49 \cdot 10^{-5}$	$6.55 \cdot 10^{-3}$
VANGUARD	$2.07 \cdot 10^{-5}$	$4.55 \cdot 10^{-3}$	$1.48 \cdot 10^{-5}$	$3.38 \cdot 10^{-3}$	$2.49 \cdot 10^{-5}$	$4.13 \cdot 10^{-3}$	$2.51 \cdot 10^{-5}$	$3.88 \cdot 10^{-3}$

Table 10 (Daily Volatility Prediction, 72nd day into future)

	Echo State Network		GARCH		LSTM		GRU	
Dataset	MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE
Amazon	$1.66 \cdot 10^{-6}$	$1.29 \cdot 10^{-3}$	$6.05 \cdot 10^{-5}$	$6.39 \cdot 10^{-3}$	$1.53 \cdot 10^{-4}$	$9.09 \cdot 10^{-3}$	$1.28 \cdot 10^{-4}$	$9.43 \cdot 10^{-3}$
Google	$9.99 \cdot 10^{-5}$	$1.00 \cdot 10^{-2}$	$3.62 \cdot 10^{-5}$	$5.05 \cdot 10^{-3}$	$1.19 \cdot 10^{-4}$	$7.05 \cdot 10^{-3}$	$1.23 \cdot 10^{-4}$	$8.52 \cdot 10^{-3}$
Apple	$6.61 \cdot 10^{-5}$	$8.13 \cdot 10^{-3}$	$5.25 \cdot 10^{-5}$	$6.17 \cdot 10^{-3}$	$1.19 \cdot 10^{-4}$	$8.97 \cdot 10^{-3}$	$1.50 \cdot 10^{-4}$	$1.07 \cdot 10^{-2}$
NASDAQ	$2.33 \cdot 10^{-4}$	$1.53 \cdot 10^{-2}$	$2.12 \cdot 10^{-5}$	$3.79 \cdot 10^{-3}$	$4.97 \cdot 10^{-5}$	$5.42 \cdot 10^{-3}$	$6.44 \cdot 10^{-5}$	$6.81 \cdot 10^{-3}$
SP500	$1.38 \cdot 10^{-4}$	$1.18 \cdot 10^{-2}$	$1.61 \cdot 10^{-5}$	$3.24 \cdot 10^{-3}$	$4.83 \cdot 10^{-5}$	$6.09 \cdot 10^{-3}$	$3.81 \cdot 10^{-5}$	$5.17 \cdot 10^{-3}$
NYSE	$9.62 \cdot 10^{-5}$	$9.81 \cdot 10^{-3}$	$1.80 \cdot 10^{-5}$	$3.37 \cdot 10^{-3}$	$2.85 \cdot 10^{-5}$	$4.54 \cdot 10^{-3}$	$2.74 \cdot 10^{-5}$	$4.21 \cdot 10^{-3}$
ISHARES	$3.10 \cdot 10^{-4}$	$1.76 \cdot 10^{-2}$	$3.31 \cdot 10^{-5}$	$4.77 \cdot 10^{-3}$	$5.76 \cdot 10^{-5}$	$5.83 \cdot 10^{-3}$	$6.66 \cdot 10^{-5}$	$5.64 \cdot 10^{-3}$
QQQ	$1.82 \cdot 10^{-4}$	$1.35 \cdot 10^{-2}$	$2.62 \cdot 10^{-5}$	$4.33 \cdot 10^{-3}$	$6.11 \cdot 10^{-5}$	$5.52 \cdot 10^{-3}$	$5.66 \cdot 10^{-5}$	$6.05 \cdot 10^{-3}$
VANGUARD	$3.16 \cdot 10^{-4}$	$1.78 \cdot 10^{-2}$	$1.34 \cdot 10^{-5}$	$2.95 \cdot 10^{-3}$	$2.38 \cdot 10^{-5}$	$3.89 \cdot 10^{-3}$	$2.37 \cdot 10^{-5}$	$3.72 \cdot 10^{-3}$

Table 11 (Log Returns Prediction, 1st day into future)

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	Echo State Network		GARCH		LSTM		GRU	
Dataset	MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE
Amazon	$7.65 \cdot 10^{-4}$	$2.77 \cdot 10^{-2}$	$2.94 \cdot 10^{-4}$	$1.71 \cdot 10^{-2}$	$2.26 \cdot 10^{-5}$	$4.75 \cdot 10^{-3}$	$1.13 \cdot 10^{-3}$	$3.36 \cdot 10^{-2}$
Google	$8.83 \cdot 10^{-5}$	$9.40 \cdot 10^{-3}$	$6.75 \cdot 10^{-5}$	$8.22 \cdot 10^{-3}$	$1.38 \cdot 10^{-5}$	$3.71 \cdot 10^{-3}$	$7.73 \cdot 10^{-4}$	$2.78 \cdot 10^{-2}$
Apple	$1.49 \cdot 10^{-3}$	$3.86 \cdot 10^{-2}$	$6.05 \cdot 10^{-5}$	$7.78 \cdot 10^{-3}$	$5.32 \cdot 10^{-5}$	$7.30 \cdot 10^{-3}$	$2.34 \cdot 10^{-4}$	$1.53 \cdot 10^{-2}$
NASDAQ	$2.76 \cdot 10^{-4}$	$1.66 \cdot 10^{-2}$	$4.62 \cdot 10^{-5}$	$6.80 \cdot 10^{-3}$	$5.12 \cdot 10^{-6}$	$2.26 \cdot 10^{-3}$	$3.24 \cdot 10^{-4}$	$1.80 \cdot 10^{-2}$
SP500	$9.06 \cdot 10^{-8}$	$3.01 \cdot 10^{-4}$	$2.27 \cdot 10^{-5}$	$4.76 \cdot 10^{-3}$	$5.93 \cdot 10^{-6}$	$2.44 \cdot 10^{-3}$	$2.30 \cdot 10^{-4}$	$1.52 \cdot 10^{-2}$
NYSE	$1.60 \cdot 10^{-5}$	$4.00 \cdot 10^{-3}$	$6.37 \cdot 10^{-6}$	$2.52 \cdot 10^{-3}$	$7.43 \cdot 10^{-6}$	$2.73 \cdot 10^{-3}$	$1.89 \cdot 10^{-4}$	$1.37 \cdot 10^{-2}$
ISHARES	$5.71 \cdot 10^{-5}$	$7.56 \cdot 10^{-3}$	$9.72 \cdot 10^{-5}$	$9.86 \cdot 10^{-3}$	$6.72 \cdot 10^{-5}$	$8.20 \cdot 10^{-3}$	$9.54 \cdot 10^{-4}$	$3.09 \cdot 10^{-2}$
QQQ	$2.65 \cdot 10^{-4}$	$1.63 \cdot 10^{-2}$	$4.77 \cdot 10^{-5}$	$6.91 \cdot 10^{-3}$	$7.15 \cdot 10^{-5}$	$8.45 \cdot 10^{-3}$	$5.22 \cdot 10^{-4}$	$2.28 \cdot 10^{-2}$
VANGUARD	$7.06 \cdot 10^{-6}$	$2.66 \cdot 10^{-3}$	$7.66 \cdot 10^{-6}$	$2.77 \cdot 10^{-3}$	$6.12 \cdot 10^{-5}$	$7.82 \cdot 10^{-3}$	$1.80 \cdot 10^{-4}$	$1.34 \cdot 10^{-2}$

Table 12 (Log Returns Prediction, 5th day into future)

	Echo State Network		GARCH		LSTM		GRU	
Dataset	MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE
Amazon	$1.37 \cdot 10^{-3}$	$3.70 \cdot 10^{-2}$	$1.51 \cdot 10^{-4}$	$1.08 \cdot 10^{-2}$	$5.04 \cdot 10^{-4}$	$1.83 \cdot 10^{-2}$	$3.90 \cdot 10^{-4}$	$1.66 \cdot 10^{-2}$
Google	$9.18 \cdot 10^{-5}$	$9.58 \cdot 10^{-3}$	$2.15 \cdot 10^{-4}$	$1.34 \cdot 10^{-2}$	$2.82 \cdot 10^{-4}$	$1.46 \cdot 10^{-2}$	$3.31 \cdot 10^{-4}$	$1.61 \cdot 10^{-2}$

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Apple	$2.58 \cdot 10^{-4}$	$1.61 \cdot 10^{-2}$	$1.14 \cdot 10^{-4}$	$8.76 \cdot 10^{-3}$	$3.17 \cdot 10^{-4}$	$1.55 \cdot 10^{-2}$	$2.94 \cdot 10^{-4}$	$1.47 \cdot 10^{-2}$
NASDAQ	$3.08 \cdot 10^{-5}$	$5.55 \cdot 10^{-3}$	$3.39 \cdot 10^{-5}$	$5.22 \cdot 10^{-3}$	$2.03 \cdot 10^{-4}$	$1.21 \cdot 10^{-2}$	$1.73 \cdot 10^{-4}$	$1.08 \cdot 10^{-2}$
SP500	$5.78 \cdot 10^{-7}$	$7.60 \cdot 10^{-4}$	$1.95 \cdot 10^{-5}$	$4.08 \cdot 10^{-3}$	$1.37 \cdot 10^{-4}$	$9.45 \cdot 10^{-3}$	$1.19 \cdot 10^{-4}$	$9.37 \cdot 10^{-3}$
NYSE	$1.05 \cdot 10^{-5}$	$3.24 \cdot 10^{-3}$	$4.48 \cdot 10^{-5}$	$5.98 \cdot 10^{-3}$	$1.32 \cdot 10^{-4}$	$8.79 \cdot 10^{-3}$	$7.50 \cdot 10^{-5}$	$7.18 \cdot 10^{-3}$
ISHARES	$6.75 \cdot 10^{-6}$	$2.60 \cdot 10^{-3}$	$6.50 \cdot 10^{-5}$	$6.61 \cdot 10^{-3}$	$2.98 \cdot 10^{-4}$	$1.53 \cdot 10^{-2}$	$2.88 \cdot 10^{-4}$	$1.47 \cdot 10^{-2}$
QQQ	$6.77 \cdot 10^{-5}$	$8.23 \cdot 10^{-3}$	$3.50 \cdot 10^{-5}$	$4.82 \cdot 10^{-3}$	$2.65 \cdot 10^{-4}$	$1.44 \cdot 10^{-2}$	$1.77 \cdot 10^{-4}$	$1.16 \cdot 10^{-2}$
VANGUARD	$9.85 \cdot 10^{-6}$	$3.14 \cdot 10^{-3}$	$4.34 \cdot 10^{-5}$	$5.60 \cdot 10^{-3}$	$2.08 \cdot 10^{-4}$	$1.19 \cdot 10^{-2}$	$9.49 \cdot 10^{-5}$	$8.25 \cdot 10^{-3}$

Table 13 (Log Returns Prediction, 18th day into future)

	Echo State Network		GARCH		LSTM		GRU	
Dataset	MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE
Amazon	$1.34 \cdot 10^{-3}$	$3.66 \cdot 10^{-2}$	$1.67 \cdot 10^{-4}$	$1.11 \cdot 10^{-2}$	$4.97 \cdot 10^{-4}$	$1.88 \cdot 10^{-2}$	$5.04 \cdot 10^{-4}$	$1.91 \cdot 10^{-2}$
Google	$1.44 \cdot 10^{-4}$	$1.20 \cdot 10^{-2}$	$1.47 \cdot 10^{-4}$	$1.06 \cdot 10^{-2}$	$3.31 \cdot 10^{-4}$	$1.51 \cdot 10^{-2}$	$3.95 \cdot 10^{-4}$	$1.67 \cdot 10^{-2}$
Apple	$3.75 \cdot 10^{-4}$	$1.94 \cdot 10^{-2}$	$8.69 \cdot 10^{-5}$	$7.40 \cdot 10^{-3}$	$4.84 \cdot 10^{-4}$	$1.82 \cdot 10^{-2}$	$3.80 \cdot 10^{-4}$	$1.58 \cdot 10^{-2}$
NASDAQ	$1.13 \cdot 10^{-4}$	$1.06 \cdot 10^{-2}$	$6.90 \cdot 10^{-5}$	$5.59 \cdot 10^{-3}$	$2.15 \cdot 10^{-4}$	$1.23 \cdot 10^{-2}$	$2.16 \cdot 10^{-4}$	$1.20 \cdot 10^{-2}$
SP500	$2.76 \cdot 10^{-6}$	$1.66 \cdot 10^{-3}$	$3.97 \cdot 10^{-5}$	$4.10 \cdot 10^{-3}$	$1.43 \cdot 10^{-4}$	$1.01 \cdot 10^{-2}$	$1.32 \cdot 10^{-4}$	$9.60 \cdot 10^{-3}$
NYSE	$1.36 \cdot 10^{-4}$	$1.17 \cdot 10^{-2}$	$4.66 \cdot 10^{-5}$	$4.94 \cdot 10^{-3}$	$1.22 \cdot 10^{-4}$	$9.03 \cdot 10^{-3}$	$1.03 \cdot 10^{-4}$	$8.45 \cdot 10^{-3}$

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ISHARES	$2.17 \cdot 10^{-6}$	$1.47 \cdot 10^{-3}$	$9.46 \cdot 10^{-5}$	$6.57 \cdot 10^{-3}$	$3.06 \cdot 10^{-4}$	$1.48 \cdot 10^{-2}$	$3.56 \cdot 10^{-4}$	$1.57 \cdot 10^{-2}$
QQQ	$1.44 \cdot 10^{-4}$	$1.20 \cdot 10^{-2}$	$6.90 \cdot 10^{-5}$	$5.56 \cdot 10^{-3}$	$2.95 \cdot 10^{-4}$	$1.45 \cdot 10^{-2}$	$2.39 \cdot 10^{-4}$	$1.27 \cdot 10^{-2}$
VANGUARD	$3.48 \cdot 10^{-5}$	$5.90 \cdot 10^{-3}$	$4.41 \cdot 10^{-5}$	$4.93 \cdot 10^{-3}$	$1.79 \cdot 10^{-4}$	$1.12 \cdot 10^{-2}$	$8.54 \cdot 10^{-5}$	$7.76 \cdot 10^{-3}$

Table 14 (Log Returns Prediction, 30th day into future)

	Echo State Network		GARCH		LSTM		GRU	
Dataset	MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE
Amazon	$2.72 \cdot 10^{-5}$	$5.21 \cdot 10^{-3}$	$4.59 \cdot 10^{-4}$	$1.53 \cdot 10^{-2}$	$4.83 \cdot 10^{-4}$	$1.88 \cdot 10^{-2}$	$4.83 \cdot 10^{-4}$	$1.87 \cdot 10^{-2}$
Google	$8.44 \cdot 10^{-4}$	$2.91 \cdot 10^{-2}$	$4.96 \cdot 10^{-4}$	$1.38 \cdot 10^{-2}$	$3.91 \cdot 10^{-4}$	$1.47 \cdot 10^{-2}$	$3.90 \cdot 10^{-4}$	$1.49 \cdot 10^{-2}$
Apple	$2.18 \cdot 10^{-4}$	$1.48 \cdot 10^{-2}$	$1.23 \cdot 10^{-4}$	$8.93 \cdot 10^{-3}$	$4.51 \cdot 10^{-4}$	$1.69 \cdot 10^{-2}$	$4.52 \cdot 10^{-4}$	$1.69 \cdot 10^{-2}$
NASDAQ	$2.61 \cdot 10^{-5}$	$5.11 \cdot 10^{-3}$	$1.08 \cdot 10^{-4}$	$7.58 \cdot 10^{-3}$	$2.51 \cdot 10^{-4}$	$1.25 \cdot 10^{-2}$	$2.51 \cdot 10^{-4}$	$1.23 \cdot 10^{-2}$
SP500	$6.18 \cdot 10^{-7}$	$7.86 \cdot 10^{-4}$	$6.21 \cdot 10^{-5}$	$5.74 \cdot 10^{-3}$	$1.53 \cdot 10^{-4}$	$9.43 \cdot 10^{-3}$	$1.54 \cdot 10^{-4}$	$9.45 \cdot 10^{-3}$
NYSE	$2.22 \cdot 10^{-6}$	$1.49 \cdot 10^{-3}$	$6.21 \cdot 10^{-5}$	$6.08 \cdot 10^{-3}$	$1.14 \cdot 10^{-4}$	$8.68 \cdot 10^{-3}$	$1.13 \cdot 10^{-4}$	$8.65 \cdot 10^{-3}$
ISHARES	$9.60 \cdot 10^{-5}$	$9.80 \cdot 10^{-3}$	$1.42 \cdot 10^{-4}$	$8.62 \cdot 10^{-3}$	$3.35 \cdot 10^{-4}$	$1.45 \cdot 10^{-2}$	$3.41 \cdot 10^{-4}$	$1.43 \cdot 10^{-2}$
QQQ	$1.04 \cdot 10^{-5}$	$3.23 \cdot 10^{-3}$	$1.09 \cdot 10^{-4}$	$7.60 \cdot 10^{-3}$	$2.60 \cdot 10^{-4}$	$1.25 \cdot 10^{-2}$	$2.60 \cdot 10^{-4}$	$1.25 \cdot 10^{-2}$
VANGUARD	$8.39 \cdot 10^{-5}$	$9.16 \cdot 10^{-3}$	$6.20 \cdot 10^{-5}$	$6.19 \cdot 10^{-3}$	$9.72 \cdot 10^{-5}$	$8.02 \cdot 10^{-3}$	$9.76 \cdot 10^{-5}$	$7.89 \cdot 10^{-3}$

Table 15 (Log Returns Prediction, 72nd day into future)

	Echo State Network		GARCH		LSTM		GRU	
Dataset	MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE
Amazon	$1.83 \cdot 10^{-4}$	$1.28 \cdot 10^{-2}$	$3.91 \cdot 10^{-4}$	$1.48 \cdot 10^{-2}$	$6.09 \cdot 10^{-4}$	$1.91 \cdot 10^{-2}$	$6.14 \cdot 10^{-4}$	$1.92 \cdot 10^{-2}$
Google	$6.69 \cdot 10^{-4}$	$2.59 \cdot 10^{-2}$	$3.08 \cdot 10^{-4}$	$1.20 \cdot 10^{-2}$	$5.80 \cdot 10^{-4}$	$1.79 \cdot 10^{-2}$	$5.72 \cdot 10^{-4}$	$1.79 \cdot 10^{-2}$
Apple	$1.02 \cdot 10^{-4}$	$3.19 \cdot 10^{-3}$	$1.48 \cdot 10^{-4}$	$9.38 \cdot 10^{-3}$	$3.62 \cdot 10^{-4}$	$1.53 \cdot 10^{-2}$	$3.50 \cdot 10^{-4}$	$1.51 \cdot 10^{-2}$
NASDAQ	$6.03 \cdot 10^{-4}$	$2.46 \cdot 10^{-2}$	$1.03 \cdot 10^{-4}$	$7.86 \cdot 10^{-3}$	$2.25 \cdot 10^{-4}$	$1.23 \cdot 10^{-2}$	$2.26 \cdot 10^{-4}$	$1.23 \cdot 10^{-2}$
SP500	$1.68 \cdot 10^{-4}$	$1.30 \cdot 10^{-2}$	$6.13 \cdot 10^{-5}$	$6.04 \cdot 10^{-3}$	$1.27 \cdot 10^{-4}$	$9.02 \cdot 10^{-3}$	$1.28 \cdot 10^{-4}$	$9.06 \cdot 10^{-3}$
NYSE	$3.41 \cdot 10^{-5}$	$1.85 \cdot 10^{-3}$	$5.77 \cdot 10^{-5}$	$6.00 \cdot 10^{-3}$	$8.92 \cdot 10^{-5}$	$7.53 \cdot 10^{-3}$	$8.81 \cdot 10^{-5}$	$7.46 \cdot 10^{-3}$
ISHARES	$5.19 \cdot 10^{-4}$	$2.28 \cdot 10^{-2}$	$1.36 \cdot 10^{-4}$	$8.88 \cdot 10^{-3}$	$2.92 \cdot 10^{-4}$	$1.40 \cdot 10^{-2}$	$3.09 \cdot 10^{-4}$	$1.43 \cdot 10^{-2}$
QQQ	$5.57 \cdot 10^{-4}$	$2.36 \cdot 10^{-2}$	$1.04 \cdot 10^{-4}$	$7.84 \cdot 10^{-3}$	$2.38 \cdot 10^{-4}$	$1.25 \cdot 10^{-2}$	$2.37 \cdot 10^{-4}$	$1.25 \cdot 10^{-2}$
VANGUARD	$1.53 \cdot 10^{-5}$	$1.24 \cdot 10^{-3}$	$5.24 \cdot 10^{-5}$	$5.83 \cdot 10^{-3}$	$7.91 \cdot 10^{-5}$	$7.01 \cdot 10^{-3}$	$8.04 \cdot 10^{-5}$	$7.03 \cdot 10^{-3}$