

Using Self-Assessment and Remediation to Raise Student Achievement in Mathematics

Prathima Prakash and John Leddo

MyEdMaster, LLC
Virginia, USA

DOI: 10.46609/IJSSER.2025.v10i01.027 URL: <https://doi.org/10.46609/IJSSER.2025.v10i01.027>

Received: 10 Jan. 2025 / Accepted: 26 Jan. 2025 / Published: 31 Jan. 2025

ABSTRACT

Cognitive Structure Analysis (CSA) is an educational framework designed to help students identify and address knowledge deficits through self-assessment, enabling them to remediate gaps in understanding. Previous studies have demonstrated the reliability of teaching students to use CSA to assess their own knowledge in various academic disciplines, including calculus (Cynkin and Leddo, 2023) and chemistry (Dandemraju, Dandemraju, and Leddo, 2024). These studies, however, primarily focused on the identification of knowledge gaps rather than their remediation. As accurate assessment does not inherently address deficiencies, later studies began to investigate CSA's role in addressing the gap. Ravi and Leddo (2024) conducted a study in which students learned an advanced chemistry topic by watching a video. Half of the students rewatched to reinforce their understanding, while the other half were trained to use CSA to self-assess their knowledge and then rewatched the video specifically to remediate assessed knowledge gaps. The CSA-trained group outperformed the control group by 15 points (1.5 letter grades) on a post-test. Similarly, Nehra and Leddo (2024) replicated this approach in Spanish instruction, finding that CSA-trained students scored an average of 25 percentage points (2.5 letter grades) higher than those who simply reread the material without self assessing. Prakash and Leddo (2025, in press) built on the findings of Ravi and Leddo (2024) and Nehra and Leddo (2024) by investigating CSA's applicability to reading comprehension; post-test results displayed that the CSA-trained group scored an average of 93%, outperforming the control group's 69%. This study builds on prior research by investigating the applicability of CSA in learning Bayes' Theorem, a foundational concept in probability theory and statistics. Twenty high school students were divided into two groups. Both groups studied Bayes' Theorem from a provided instructional document, but only the experimental group used CSA to self-assess their knowledge and remediate gaps. Post-test results revealed that the experimental group significantly outperformed the control group, scoring an average of 85.5% compared to the

control group's 58.5%. These findings underscore CSA's potential to improve understanding of abstract mathematical concepts while fostering self-directed learning.

INTRODUCTION

Throughout history, assessment has served as a measure of students' learning. Traditionally, "learning" has been defined by the number of correct answers on tests, as per classical test theory, which assumes that a student's total correct responses reflect their knowledge level (de Ayala, 2009).

Assessment methods typically fall into two categories: selecting correct answers from choices or constructing answers independently. Multiple-choice tests, widely used for their efficiency in grading, allow for guessing, which can inflate scores (Chaoui, 2011; Elbrink and Waits, 1970; O'Neil and Brown, 1997). Constructive response tests require students to provide their own answers, encouraging logical reasoning and offering a more accurate measure of knowledge (Herman et al., 1944; Frary, 1985). However, both methods rely on the assumption that correct answers signify learning. This assumption is problematic, as incorrect answers may point to underlying knowledge gaps, while correct answers might result from memorization or guessing, not true understanding.

Cognitive Structure Analysis (CSA) is an assessment method designed to uncover the underlying knowledge concepts a student possesses, identifying the source of errors for targeted remediation (Leddo et al., 2022; Ahmad and Leddo, 2023; Zhou and Leddo, 2023; Dandemraju, Dandemraju, and Leddo, 2024). CSA is rooted in cognitive psychology research, which identifies various knowledge types, such as semantic nets (Quillian, 1966), production rules (Newell and Simon, 1972), scripts (Schank and Abelson, 1977), and mental models (de Kleer and Brown, 1981). Together, these form the INKS framework (Integrated Knowledge Structure), developed by John Leddo (Leddo et al., 1990). This framework suggests that expert knowledge is organized around scripts and principles that enable predictions and explanations.

CSA, which integrates INKS principles, has shown strong correlations with problem-solving performance: 0.966 in Algebra 1 (Leddo et al., 2022), 0.63 in scientific method problem-solving (Ahmad and Leddo, 2023), and 0.80 in precalculus (Zhou and Leddo, 2023). By assessing students' conceptual understanding, CSA enables educators to address knowledge gaps effectively, leading to significant improvements in student performance (Leddo and Ahmad, 2024).

Although CSA has proven effective, the responsibility for diagnosing and remediating students' knowledge gaps lies primarily with teachers, who often manage large numbers of students. Teaching students to self-assess their knowledge could alleviate this burden. Unlike self-

explanation (Chi et al., 1989), which involves generating explanations for learned material, self-assessment involves evaluating one's knowledge after learning.

Cynkin and Leddo (2023) demonstrated that high school calculus students could accurately self-assess their knowledge using CSA, while Dandemraju, Dandemraju, and Leddo (2024) extended this finding to chemistry. These studies, however, addressed only the identification of knowledge gaps, not their remediation. Accurate assessment does not equate to addressing deficiencies, just as diagnosing a medical issue does not equate to treating it.

To address this issue, Ravi and Leddo (2024) conducted a study in which students learned an advanced topic in chemistry by watching a video. Half the students were told to rewatch the video to fill in any knowledge gaps, while the other half were taught to self-assess their knowledge using CSA and then told to rewatch the video to fill in any assessed knowledge gaps. The group that was taught to self-assess scored 15 points or 1.5 letter grades higher on a post-test than students who simply rewatched the video without self-assessment. Nehra and Leddo (2024) replicated the Ravi and Leddo study to the learning of Spanish. They found that students performing self-assessment plus remediation scored, on average, 25 percentage points or 2.5 letter grades higher than those re-reading the material without performing a self-assessment. Prakash and Leddo (2025, in press) extended the Ravi and Leddo (2024) and Nehra and Leddo (2024) findings to another subject area: reading comprehension. The results revealed a mean post-test score of 8.3 out of 12 (69.17%) for the control group and 11.2 out of 12 (93.33%) for the experimental group. This difference in averages was statistically significant ($t = 3.75$, $df = 11.07$, $p < .01$). Notably, individual scores further illustrated the disparity: the lowest score in the control group was 41.67%, whereas the lowest in the experimental group was 83.33%. This is the difference between an F letter grade and B letter grade. This study aims to extend CSA's application to a mathematical context by examining its effectiveness in teaching math, specifically, the topic of Bayes' Theorem.

Bayes' Theorem is a probabilistic model that calculates conditional probabilities and is widely used in fields such as data science, medicine, and finance. Despite its significance, students often struggle to understand its principles due to the abstract nature of the concepts involved. This study investigates whether CSA can help students self-assess and remediate their understanding of Bayes' Theorem, thereby improving performance on related assessments.

METHOD

Participants

20 male and female Loudoun County Public Schools students were selected to participate in this study. All students were high school students, and they were not paid for their participation.

Materials

A Google Form for the control group with the Bayes' Theorem guide and 20 comprehension questions is provided below.

<https://docs.google.com/forms/d/e/1FAIpQLScwsD5MAFzbVx6gnWDmtbxHy4bpLIJbQOZjsfZDMY05crRXQ/viewform>

A self-assessment was created in order to help students in the experimental group re-evaluate their understanding of the content provided in the guide. It showed an example of a student self-assessing knowledge of a mathematical concept that included facts, strategies, procedures, and rationales. It was modeled after the self-assessment template previously reported in Ravi and Leddo (2024).

Self-Assessment: Math

I want to teach you how to assess your own knowledge that you have about a subject area. Let's do this by taking an example that you already know. Suppose you wanted to assess your own knowledge about solving 2-step equations of the form $ax + b = c$. An example of this type of problem is $2x + 3 = 15$. If I want to be able to solve problems like these, I need four types of knowledge. These are facts, strategies, procedures and rationales. Fact are concepts you have that describe objects or elements. For example, for two step equations, I need to know what variables, constants, coefficients, equations, and expressions are. Strategies are general processes I would use to solve a problem. For two step equations, this would be reverse order of operations. Procedures are the specific steps that I would use in a strategy. So if I am using reverse order of operations, I need to know additive and multiplicative inverses. Finally, I need to know rationales which are the reasons why the strategies or the procedures work the way they do. For example, this could include things like the subtraction or the division property of equality that says that when you do the same operation to both sides of an equation, you preserve the value of the equation. You can think of facts as telling you "what", strategies and procedures as telling you "how" and rationales as telling you "why".

With this in mind, this is how I might assess my own knowledge of solving two step equations. For facts, I need to know what variables, constants, coefficients, equations and expressions are. A variable is an unknown quantity, usually represented by a letter. A constant is a specific number. A coefficient is a number that you multiply a variable by like $2x$. An equation is an expression that is equally to another expression and the two expressions are joined by an equal sign. An expression is one or more terms that are combined by mathematical operations like addition, subtraction, multiplication and division.

For strategies, I need to know reverse order of operations which is SADMEP. This stands for subtraction, addition, division, multiplication, exponents and parentheses. I know that I'm supposed to do these in order but I don't remember whether I'm supposed to do subtraction always before addition or just which one goes first. The same is true for division and multiplication.

For procedures, I need to know additive inverse and multiplicative inverse. The additive inverse is taking the number with the opposite sign as the constant and adding it to both sides of the equation. The multiplicative inverse is taking the inverse of the coefficient of the variable and multiplying both sides of the equation by it. However, if the coefficient is negative, I'm not sure if the multiplicative inverse is supposed to be negative as well.

For rationales, I believe the two rationales I need are the subtraction property of equality and the division property of equality. The subtraction property of equality says that if I subtract the same number from both sides, which is what I'm doing with the additive inverse, I preserve the equality. Similarly, the division property of equality says that if I divide both sides of the equation by the same number, which is what I'm doing with the multiplicative inverse, I preserve the equality.

When I look over what I wrote, I see that I am good with my facts. On my strategy, I'm not sure about the order of steps in reverse order of operations when it comes to subtraction and addition or multiplication and division, so I need to learn those. On procedures, I'm not sure what to do with multiplicative inverses when the coefficient is negative, so I need to learn that as well. For rationales, I think I'm OK. I don't think I have any missing facts/concepts that I left out that I should know or I didn't list any facts/concepts where I didn't know what they were. For the strategy, I believe I listed the correct strategy and parts of the strategy, but I wasn't sure about some of the ordering of steps in the strategy. For procedures, I was good on the additive inverse but had a question on carrying out the multiplicative inverse when the coefficient was negative. For rationales, I think I had all the rationales that were important and that I understood them as well. I don't think I left anything out.

A Google Form for the experimental group with the Bayes' Theorem guide, math self-assessment, and 20 comprehension questions is provided below.

https://docs.google.com/forms/d/e/1FAIpQLSez6QDC5_Xp4ohJMYeDOm39WR0ELh8pJbAHMJZxfH5rXLpMNA/formResponse

In addition to the math assessment, an answer key was created in order to evaluate each participant's answer to the math question. There was no partial credit, with 1 point for each correct response and 0 for each incorrect response.

Procedure

Participants were randomly assigned to one of two groups: control (MA1) and experimental (MA2). Both groups received a Google document explaining Bayes' Theorem, including its formula and applications. The control group was instructed to study the material and complete a post-test, with no structured guidance on how to address knowledge gaps. The experimental group was trained to use CSA for self-assessment. After studying the document, participants in the experimental group evaluated their understanding using CSA and revisited the material to address knowledge gaps before taking the same post-test as the control group. The post-test included 20 questions assessing conceptual understanding, application, and problem-solving. Participants were not permitted to access the Bayes' Theorem guide when answering the questions.

RESULTS

The participants' data were analyzed by examining the number of correct responses on the post-test. The results revealed a statistically significant difference in performance between the two groups. The control group (MA1) achieved a mean score of 58.5%, while the experimental group (MA2) scored an average of 85.5%. Statistical analysis yielded a t-value of 4.38 (df = 18, p = 0.0004), confirming the significance of the difference. Individual scores also highlighted the disparity. The control group's lowest score was 6/20 (30%), whereas the experimental group's lowest score was 15/20 (75%). The experimental group demonstrated both a higher mean and greater consistency in performance.

Additionally, 9 out of 10 participants in the experimental group, when asked, voted in favor of implementing the self-assessment system into schools, to improve mathematical understanding. This suggests that the approach is both effective and appealing for learners. In contrast, participants in the control group reported no benefits from rereading the guide, as they were simply given a description and examples of the concept without structured tools to identify and address their knowledge gaps effectively.

DISCUSSION

This study aimed to evaluate the effectiveness of Cognitive Structure Analysis (CSA) in helping high school students self-assess and remediate knowledge gaps in mathematics. The findings reaffirm CSA's utility in addressing knowledge deficiencies, as evidenced by the experimental group's 27-point advantage over the control group. These results align with prior studies, such as Nehra and Leddo's (2024) work on Spanish learning, which reported significant gains through CSA, and Ravi and Leddo's (2024) chemistry study, which found a 15-point improvement. However, this study goes further by demonstrating CSA's particular efficacy in mathematics—a

discipline fundamental to a wide range of academic and professional fields (Leddo, Ahmad, & Zhou, 2022).

Notably, the 27-point improvement observed in this study exceeds the gains reported in previous CSA research, suggesting unique advantages of applying CSA to mathematics. Mathematics, given its structured nature and emphasis on problem-solving, allows students to isolate and address specific errors in reasoning or calculation more readily than in disciplines like chemistry or reading comprehension. This specificity may enable more targeted remediation, amplifying the effectiveness of CSA in promoting mastery.

The implications of these findings extend well beyond the immediate context of this study. The current U.S. educational system, often criticized for its one-size-fits-all approach, places an immense burden on teachers to diagnose and address the unique learning gaps of each student. With many classrooms operating at a high student-to-teacher ratio, individualized attention is often impractical. CSA offers a scalable solution by equipping students with the tools to identify and address their own deficiencies, reducing reliance on teacher intervention.

The psychological benefits of CSA are equally significant. Participants in the experimental group reported not only better comprehension of Bayes' Theorem but also greater confidence in their ability to tackle similar challenges. This aligns with Nehra and Leddo's (2024) findings that self-assessment builds self-efficacy, a critical component of long-term academic and professional success. Confidence in one's ability to learn and adapt is crucial in combating the fixed mindset that often inhibits growth in mathematics and other STEM fields.

From a societal perspective, widespread adoption of CSA could have transformative effects on educational equity. By providing students with a method to independently assess and improve their knowledge, CSA could help bridge achievement gaps, particularly for students in under-resourced schools where access to one-on-one teacher support is limited. Additionally, CSA has the potential to create a culture of proactive learning, where students view mistakes not as failures but as opportunities for growth, a mindset that is critical for success in both academic and real-world contexts.

Future research should explore CSA's applicability to other mathematical concepts, such as calculus, linear algebra, or statistics, as well as its utility in addressing more abstract or multi-step problems. Longitudinal studies could also investigate the lasting impact of CSA on students' academic trajectories, including their performance in advanced coursework or standardized testing. Additionally, integrating CSA into existing curricula could provide a systematic and sustainable approach to fostering independent learning across diverse educational settings. Such

integration could be particularly impactful in large-scale educational systems, where resources are often spread thin and teacher-led remediation is not feasible.

In conclusion, this study not only reinforces the effectiveness of CSA in mathematics but also underscores its broader potential to transform educational practices, enhance student confidence, and address systemic challenges in education. By providing students with the tools to take ownership of their learning, CSA holds the promise of creating a more equitable educational system that prepares learners for the demands of the future.

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