

COMBINED USE OF DATA DEVELOPMENT ANALYSIS AND GREY RELATIONAL ANALYSIS IN THE EFFICIENCY ANALYSIS OF PUBLIC HOSPITALS IN THE METROPOLITAN CITIES OF TURKEY

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ABSTRACT

The objective of this study was to determine the efficiencies of public hospitals affiliated with the Ministry of Health - Turkish Institute of Public Hospitals at the provincial level, using Data Envelopment Analysis and ranking efficient hospitals and important efficiency factors using Grey Relational Analysis. In this study, a Data Envelopment Analysis model targeting maximization was used to analyse the efficiencies of 30 metropolitan municipality hospitals using 4 inputs (number of beds, numbers of surgeons and general practitioners, expenditures made from circulating capital) and 3 outputs (number of patients examined in the emergency service, bed occupancy rates and total number of A+B+C group operations). As a result of the Data Envelopment Analysis, public hospitals in 16 cities were found to be efficient, and 14 hospitals were found to be inefficient at the provincial level. Based on the outcomes of the Grey Relational Analysis used to rank the performances of hospitals within 14 cities efficient at the provincial level, hospitals in Istanbul, Gaziantep and Ankara shared the top 3 places. Among variables included in the analyses, the most effective variables for predicting the performances of the hospitals were expenditures made from annual circulating capitals, bed occupancy rates and the number of general practitioners per hospital.

Keywords: Efficiency; Data Envelopment Analysis; Grey Relational Analysis; public hospitals.

INTRODUCTION

Worldwide globalization and new concepts in public organization have exerted their impacts on the health sector. The effects of globalization on the health sector manifest themselves through institutional, financial, sociocultural and environmental dimensions. Within this concept, the role

played by the government in health care services has also begun to change. As a result of emerging developments in many countries, hospitals are gaining administrative and financial autonomy, in other words, governments allow for the implementation of decentralization in health care services, which constitutes the main topic of health sector reform in many European countries. These practices aim for the effective distribution of resources according to the requirements and people's participation in the decision-making processes related to health care services.

In parallel with developments in Turkey, new regulations have been implemented in the delivery of health care services in countries across the globe. Eighty-eight "Associations of Public Hospitals" were established at the provincial level in Turkey. By means of these associations, the resources of all public hospitals affiliated with the Ministry of Health have been planned and managed. The new concept of management, aims to grant autonomy to hospitals and offer accessible, efficient, effective and high-quality health care services suitable to the needs and expectations of the citizens.

For the effective use of the limited resources of public hospitals, it is necessary to assess the current efficiencies of hospitals and determine the steps that need to be taken to improve efficiency where needed. The most frequently used method in the measurement of efficiency is Data Envelopment Analysis (DEA). DEA has become a frequently used benchmarking criterion in various fields, including hospitals, universities, manufacturing plants, logistic firms, banks and sports. Additionally, the Grey Relational Analysis (GRA) has been increasingly reported in the literature as a decision – making method. This analytical decision-making tool has taken its place in the literature under the main heading of the Grey System Theory.

The definition of outputs of public hospitals differs according to their relevant purposes of production. Hospitals produce very diverse outputs, as measured by the numbers of outpatient examinations, operations, days of hospitalization, childbirths, and bed occupancy rates. Therefore, DEA, which measures efficiency using more than one input and output regardless of diversities among measurement units, is a preferred method for assessing the activities performed in public hospitals [1]. Although DEA provides important information about the improvements required for inefficient units to become efficient, it does not rank those units with established efficiencies. Thus, in many studies cited in the literature, GRA has been used for ranking the performances of efficient units, as determined via DEA [2-4]. The DEA technique has been successfully used in many research studies within the health sector [5-11]. In recent years, DEA has increasingly been used in studies concerning efficiency analysis in Turkey [12-22].

This study, aims to determine and analyse the efficiencies of hospitals in metropolitan cities affiliated with the Turkish Institute of Public Hospitals at the provincial level. It is based on data from the Associations of Public Hospitals 2014, prepared by the Directorate of the Department of Statistics, Analysis and Reporting in the Republic of Turkey Ministry of Health - Turkish Institute of Public Hospitals [23]. Within the scope of the study, efficiencies of public hospitals in 30 cities with metropolitan municipalities were analysed at the provincial level via DEA. The analysis targeted profit maximization using 4 inputs (number of beds, surgeons and general practitioners in clinics and annual expenditures made from circulating capital) and 3 outputs (number of examination in the emergency service, bed occupancy rate and total number of operations (A+B+C)). GRA was used to rank the performances of the cities with efficient public hospitals. As a result of the GRA, the factors effecting the performances of the efficient cities were determined and ranked according to the impact of their effects. We hypothesize that this study, performed at the provincial level, will provide important information for the relevant parties (Ministry of Health, Hospital Management).

METHODS

In this study, DEA was used in the analysis of the efficiencies of public hospitals at the provincial level in 30 metropolitan municipalities. GRA was used to rank both the efficient public hospitals, as determined by the DEA and the input-output variables utilized in the analysis by degree of efficiency.

Data Envelopment Analysis (DEA)

DEA is a measurement technique with a linear programming base that measures the relative efficiencies of DMUs in productive conditions with more than one input and output that can-not be expressed as a single input or output [16]. DEA models have been analysed in two groups: those measuring efficiencies of inputs and those measuring efficiencies of outputs. DEA models measuring efficiencies of inputs investigate the most optimal input combination to produce a certain output combination. The complete DEA model has the advantage of easily accommodating multiple inputs and easily producing, multiple outputs, and it does not make behavioural assumptions about the DMUs [24].

DEA has several advantages over other forms of data analysis. First, it does not impose a specific functional form on the production function. Second, it mitigates the endogeneity bias that may be associated with traditional regression techniques. Last, as noted and, unlike other methods such as ratio and regression analysis, DEA allows for the identification of improvement targets for inefficient units, providing useful insights about the sources of inefficiency. From this perspective, this nonparametric approach is valuable, because it can help entrepreneurs and

policy makers understand if they are processing their inputs, correctly, and, it indicates best practices for improving efficiency [25].

In large and complicated datasets, using a standardized process could facilitate performance assessment and help to (1) translate the aim of the performance measurement into a series of small tasks, (2) select homogeneous DMUs and suggest an appropriate input/output selection, (3) detect a suitable model, (4) provide means for evaluating the effectiveness of the results and (5) suggest a proper solution to improve the efficiency and productivity of entities (also called DMUs) [26].

In this study, the DMUs in the DEA application are public hospitals in metropolitan cities because they produce similar outputs using similar inputs. Explanations concerning how the 4 inputs and 3 outputs in the DEA model of this study were analysed are as follows:

During the determination process to decide the variables that were to be used in the study, relevant literature was screened, and generally accepted variables (used in DEA models from previous studies) were included in this study to facilitate fair comparison with similar studies. From among the provincial data that were derived from relevant studies, the total number of beds in the clinics, numbers of surgeons and general practitioners, and annual expenditures made from circulating capital were selected as input variables. As output variables, the number of examinations in the emergency services, bed occupancy rate, and total number of operations (A+B+C) were selected. All data about variables were gathered from Associations of Public Hospitals 2014, prepared by the Directorate of the Department of Statistics, Analysis and Reporting in the Republic of Turkey Ministry of Health Turkish Institute of Public Hospitals, from 01/01/2013 to 12/31/2013.

Number of beds in clinics: The number of beds in the clinics is the reason for existence of the hospitals and is also one of the important criteria for the efficiency of inpatient treatment. It becomes the most important criterion when considered in conjunction with bed occupancy rates, which, together, demonstrate the extent of the whole capacity of the hospital when used efficiently. Therefore, for the surgeons to operate at optimal levels, vacant hospital beds should be available. Furthermore, a linear correlation exists between the total amount of expenses and the number of beds: cost per hospital bed decreases, as the number of beds increases. In this study, the number of beds in clinical departments indicates the sum of clinic patient beds, either vacant or occupied, excluding patient beds in the intensive care unit of the provincial public hospitals.

Number of surgeons: The total number of surgeons has been evaluated as the most important criterion impacting the number of patients hospitalized in surgical clinics for clinical and surgical

indications, as well as for the total number of operations. The ratio between the total number of operations and the total number of physicians gives the mean number of operations for each physician. Therefore, it is considered to be an important variable for the demonstration of the efficiency model for surgeons.

Number of general practitioners: In Turkey, general practitioners are employed in the emergency services. A linear correlation exists between the number of general practitioners and the number of examinations performed in the emergency services: the average number of examinations in the emergency services decreases as the number of general practitioners increases. As the average number of patients for each general practitioner increases, the waiting time in the emergency service increases, resulting in an increase in the number of patients leaving without being examined, to consult another hospital.

Annual circulating capital expenditures: This indicates the sum of all annual payments made from the circulating capitals of all provincial public hospitals, as calculated at the end of the 2013 fiscal year.

Total number of examinations in the emergency service: This indicates the total number of examinations performed in the emergency services of provincial public hospitals.

The bed occupancy rate is a ratio between the hospital capacity and the actual utilization of beds in all provincial public hospitals, excluding intensive care units. The utilization rate of hospital beds in all provincial public hospitals within a year is calculated using the following formula:

Bed occupancy rate = (Number of inpatient days x 100) / (Actual number of beds x 365 days)

Total number of operations (A+B+C): This indicates the total number of A+B+C group operations in the relevant clinics of all provincial public hospitals. The classification of operations was based on the International Classification of Health Interventions (ICHI) of The World Health Organization. In compliance with the definition provided in this classification, the number of A + B + C group operations were calculated. According to the ICHI of The World Health Organization, diagnostic interventions and small surgical operations are not included in this category.

Outcomes obtained from this study are valid for the input and output combinations used and for all data gathered within the year 2013. Since DEA is a measurement tool for relative performance and is, dependent on the input and output combinations and observation sets, outcomes can vary from year to year. Because hospitals focus on output maximization rather

than input minimization, the BCC (Banker, Charnes and Cooper) model was used with a variable yield aiming for output maximization.

In the selection of DMUs, similarities between DMUs for production technologies exist. Moreover, if we specify the number of inputs and outputs as “m” and “s”, respectively, then number of DMU in the DEA should be at least $m+s+1$. Another constraint is that the number of DMUs should be at least 2-fold higher than the total number of variables [27]. This study design was constructed with 4 input variables, and 3 output variables, and 30 DMUs. In the study, the number and characteristics of the DMUs provide both of these constraints.

Grey Relational Analysis (GRA)

Grey Relational Analysis is a method of decision-making and analysis that has taken its place in the literature under the main heading of Grey Theory [28]. Professor Julong Deng from the Hua Chung University of Science and Technology in Thailand, first proposed the Grey Theory in 1982. The Grey Theory has been applied under the subheadings of GRA, Grey modelling, Grey estimate and Grey decision-making [29].

The steps for calculating GRA are as follows [30,1]:

Step 1: The reference sequence, consisting of the number of units (m) and the number of variables (n), and the reference sequence that will be compared with this sequence are determined:

$$\begin{aligned} X_0 &= \{x_0(1), x_0(2), \dots, x_0(k)\}, \\ X_1 &= \{x_1(1), x_1(2), \dots, x_1(k)\}, \\ X_i &= \{x_i(1), x_i(2), \dots, x_i(k)\}, \\ X_N &= \{x_N(1), x_N(2), \dots, x_N(k)\} \end{aligned}$$

Step 2: Data are normalized.

Before calculating the Grey relational coefficient, data should be standardized to compare the indicators of various dimensions [31]. Based on the following steps, data are converted to their standard values. When we consider that factors come from different sources and are measured in various units, the first step in GRA is to express all data using the same unit. However, in cases where series consist of values within a very large range, these values should be drawn from within a narrow range with standardization. This normalization process is termed “Grey relational generating”. One of the most frequently used methods in normalization is the linear data pre-processing technique. In the normalization of factor series, the determination of the best

criteria that reflect the characteristics of the factor series among “the higher, the better,” and “the lower, the better” [32,33] should be taken into consideration.

If the aim of the original series is infinity, then it will carry the characteristics of “the higher, the better.” In this case, the original series is normalized, as seen in Equation 1:

$$x_i^*(j) = \frac{x_i(j) - \min_j x_i(j)}{\max_j x_i(j) - \min_j x_i(j)} \quad (1)$$

If “the lower, the better” is a characteristic of the original series, then the original series is normalized as follows:

Equation 2:

$$x_i^*(j) = \frac{\max_j x_i(j) - x_i(j)}{\max_j x_i(j) - \min_j x_i(j)} \quad (2)$$

Step 3: Calculation of absolute values [].

At, the absolute value between X0 and Xi in j units is expressed as:

Equation 3:

$$\Delta_{0i}(j) = |x_0(j) - x_i(j)| \text{ and } \Delta_{\max} = \text{Max}_i \max_j \Delta_{0i}(j), \Delta_{\min} = \text{Min}_i \min_j \Delta_{0i}(j) \quad (3)$$

Step 4: A Grey relational coefficient is calculated for the normalized series. The Grey relational coefficient for comparator Xi and X0 reference sequences is defined in j units as follows:

Equation 4:

$$\gamma_{0i}(j) = \frac{\Delta_{\min} + \Delta_{\max}}{\Delta_{0i}(j) + \Delta_{\max}} \quad (4)$$

The J coefficient in the equation is used to discard the possibility of being the most probable extreme value and is generally accepted as 0.5

Step 5: The Grey relational degree, which is the sum of the weighted Grey relational coefficients, is calculated. The Grey relational degree (GRD) for Xi series is given as:

Equation 5:

$$\Gamma_{0i} = \sum_{j=1}^K w_j \gamma_{0i}(j) \quad (5)$$

The weighted j is expressed as w_j . If the calculation of the weighted value is not necessary, then as an average.

RESULTS

The findings obtained from the DEA and GRA are explained in the following sections.

Analysis of hospital efficiencies using DEA

As a result of DEA, DMUs with 100% efficiency are determined to be efficient; for inefficient DMUs, reference groups should be constructed. The efficiency statuses of 30 hospitals affiliated with metropolitan municipalities are given in Table 1.

Table 1. Efficiency status and reference frequencies of the hospitals according to provinces.

Provinces	Efficiency scores	Efficiency status	Reference frequencies
Mardin	100	Efficient	2
Sakarya	100	Efficient	1
Erzurum	100	Efficient	1
Hatay	100	Efficient	3
Van	100	Efficient	5
Urfa	100	Efficient	7
Tekirdag	100	Efficient	3
Kocaeli	100	Efficient	0
Antalya	100	Efficient	0
Adana	100	Efficient	0
Eskişehir	100	Efficient	2
Bursa	100	Efficient	0
Denizli	100	Efficient	7
Gaziantep	100	Efficient	10
Ankara	100	Efficient	1
Istanbul	100	Efficient	3
Kahramanmaraş	99.64	Inefficient	0
Mersin	98.72	Inefficient	0
Malatya	98.28	Inefficient	0
Aydın	94.37	Inefficient	0

Kayseri	92.23	Inefficient	0
Ordu	91.89	Inefficient	0
Samsun	90.20	Inefficient	0
Izmir	89.31	Inefficient	0
Trabzon	86.20	Inefficient	0
Mugla	85.73	Inefficient	0
Manisa	85.14	Inefficient	0
Konya	85.08	Inefficient	0
Balıkesir	84.90	Inefficient	0
Diyarbakir	77.54	Inefficient	0

According to the efficiency rates in Table 1, hospitals affiliated with the Turkish Institute of Public Hospitals in 16 provinces were successful as determined by the variables included in the analysis at the provincial level, and 14 provinces were found to not be efficient concerning their performances at the provincial level. The inefficient provinces of Kahramanmaraş (99.64%), Mersin (98.72%) and Malatya (98.28%), as well as Diyarbakir (77.54%), which had the lowest efficiency rating, were also inefficient regarding the number of operations performed, bed occupancy rates and the number of patients examined in the emergency services.

One of the benefits of DEA is its ability to set and offer achievable goals for inefficient DMUs to improve their performances. It has been assumed that inefficient DMUs applying the same methods used by relatively efficient units can achieve the same level of efficiency. Although, performance improvement rates have been calculated for each of the provinces with inefficient public hospitals, as an example for comments to be made, the two cities with the highest and the lowest inefficiency rates were compared and Table 2 was constructed:

Table 2. Potential improvement rates and reference sets for provinces with inefficient public hospitals.

	Factors		Actual value	Target value	Potential improvement rate (%)	Reference group
Kahramanmaraş (99.64)	Input	Number of beds	1456	1424.84	-02.14	Denizli Erzurum Eskisehir Gaziantep Sakarya
		Number of surgeons	141	141	00.00	
		Number of general practitioners	62	62	00.00	
		Annual circulating capital	243154834	242183960	-00.40	
	Output	Number of examinations in the emergency service	1162609	1166784.67	00.36	
		Bed occupancy rates	70	70.25	00.36	
Mersin (98.72)	Input	Number of beds	1932	1848.78	-04.31	Gaziantep Sanliurfa Istanbul
		Number of surgeons	273	221.36	-18.92	
		Number of general practitioners	115	98.85	-14.04	
		Annual circulating capital	359818836	359818836	00.00	
	Output	Number of examinations in the emergency service	2194615	2222996.92	01.29	
		Bed occupancy rates	74	77.68	04.98	
Balıkesir (84.90)	Input	Number of beds	2092	1539.87	-26.39	Denizli Gaziantep Hatay Sanliurfa
		Number of surgeons	207	199.04	-03.85	
		Number of general practitioners	95	95	00.00	
		Annual circulating capital	307735985	307735985	00.00	
	Output	Number of examinations in the emergency service	1453848	1712457.10	17.79	
		Bed occupancy rates	71	83.63	17.79	
Diyarbakir (77.54)	Input	Number of beds	2135	1655.54	-22.46	Gaziantep Sanliurfa Van
		Number of surgeons	196	196	00.00	
		Number of general practitioners	109	89.40	-17.99	
		Annual circulating capital	362386935	325030224.76	-10.31	
	Output	Number of examinations in the emergency service	1520285	1960687.30	28.97	
		Bed occupancy rates	62	79.96	28.97	
		Number of operations (Group A+B+C)	3915	51713.80	1220.91	

As seen in Table 2, if public hospitals in Kahramanmaraş decrease their beds in clinical departments (2.14%) and, their annual expenditures from circulating capital (0.40%) and increase the number of examinations in their emergency services, their bed occupancy rates and the number of operations (0.36%) without changing the number of surgeons and general practitioners, then they will report successful performances just like the referenced, efficient provinces of Denizli, Erzurum, Eskisehir, Gaziantep and Sakarya. Similarly, if public hospitals in Izmir decrease their number of beds in clinical departments (15.91%), surgeons (10.38%) and general practitioners (44.45%) and increase the number of examinations in their emergency services (11.97%), their bed occupancy rates (20.63%) and their total number of operations (11.97%) without changing their expenditures made from circulating capitals, then Izmir's public hospitals will make Izmir into a successful city just like the referenced, efficient metropolitan cities of Gaziantep, Ankara and Istanbul.

The last column of Table 2 contains the units, that will be referenced by inefficient provinces. Referenced units and inefficient units, which need a reference, have the same nature of activity. Therefore, any inefficient unit can be efficient by engaging in activities similar to the activity structures of its determined referenced unit. The frequency of being a reference for insufficient units signifies the strength of the activity structure of a referenced unit. Accordingly, Gaziantep, Urfa and Denizli, which were referenced ten times, and Urfa and Denizli, which were referenced seven times, have strong activity structures. Therefore, the public hospitals of these three cities yield the greatest amount of outputs: in other words, they fulfil their activities well.

Ranking of efficient hospitals using GRA

In the second phase of the application, the performances of the efficient services determined by the DEA were evaluated using GRA. As an outcome of the GRA, both the efficient services and seven of the variables that were analysed in the DEA were ranked with respect to their efficiencies. The different stages of GRA were applied as follows:

1. Constructing the decision-making matrix

In this phase, the values of each efficient DMUs of each province with efficient public hospitals as assessed via the DEA are given in Table 3.

Table 3. Decision-making matrices of provinces.

	X1 (max)	X2 (max)	X3 (max)	X4 (min)	X5 (max)	X6 (max)	X7 (max)
Mardin	750	89	47	135713960	937778	58	22364
Sakarya	1327	165	60	257835828	1388240	68	38274
Erzurum	1693	130	65	253550058	1102119	60	43474
Hatay	1429	198	110	299267889	1759598	82	42073
Van	1152	116	51	206156808	1344215	69	26825
Urfa	1705	185	96	314469079	2208047	72	44991
Tekirdag	952	127	49	156187465	891057	71	16837
Kocaeli	2024	287	73	364090837	1946174	65	49088
Antalya	2200	375	92	419543267	2081275	75	66812
Adana	3051	337	101	482521733	1810998	76	76834
Eskisehir	1525	157	46	225837589	628806	75	38127
Bursa	3901	454	113	600953048	2789043	71	87779
Denizli	1276	151	63	212400922	1029433	80	38049
Antep	1797	233	98	375554359	1990002	90	65767
Ankara	7403	1345	223	1806016410	4016268	78	227923
Istanbul	13933	2450	333	3098613421	11093825	75	362215

2. Normalization

Following the construction of the decision-making matrix (Table 3) for the variable X4, Equation 2 (and for all other variables, Equation 1) was used to obtain normalization data for the reference series $X_0 = \{1.00, 1.00, 1.00, \dots, 1.00\}$. Normalization data are shown in Table 4.

Table 4. Normalization data.

Provinces	Variables $X_i^*(j), j=1,2,\dots,8$						
	Inputs				Outputs		
	X1	X2	X3	X4	X5	X6	X7
Reference	1.0	1.0	1.0	1.0	1.0	1.0	1.0
Mardin	0	0	0.0035	1	0.0296	0	0.0160
Sakarya	0.0438	0.0322	0.0488	0.9588	0.0726	0.3125	0.0621
Erzurum	0.0715	0.0174	0.0662	0.9602	0.0452	0.0625	0.0771
Hatay	0.0515	0.0462	0.2230	0.9448	0.1081	0.7500	0.0731
Van	0.0305	0.0114	0.0174	0.9762	0.0684	0.3438	0.0289
Urfa	0.0724	0.0407	0.1742	0.9397	0.1509	0.4375	0.0815

Tekirdag	0.0153	0.0161	0.0105	0.9931	0.0251	0.4063	0
Kocaeli	0.0966	0.0839	0.0941	0.9229	0.1259	0.2188	0.0934
Antalya	0.1100	0.1211	0.1603	0.9042	0.1388	0.5313	0.1447
Adana	0.1745	0.1050	0.1916	0.8829	0.1130	0.5625	0.1737
Eskisehir	0.0588	0.0288	0	0.9696	0	0.5313	0.0616
Bursa	0.2390	0.1546	0.2334	0.8430	0.2064	0.4063	0.2054
Denizli	0.0399	0.0263	0.0592	0.9741	0.0383	0.6875	0.0614
Antep	0.0794	0.0610	0.1812	0.9191	0.1301	1	0.1417
Ankara	0.5047	0.5320	0.6167	0.4363	0.3237	0.6250	0.6112
Istanbul	1	1	1	0	1	0.5313	1

3. Calculation of absolute values []

is the difference between each value and its reference value. Equation 3 was used to calculate absolute values. Absolute values are shown in Table 5.

Table 5. Absolute values $\Delta_{0i}(j)$.

Provinces	Variables $X_i^*(j), j=1,2,\dots,8$						
	Inputs				Outputs		
	X1	X2	X3	X4	X5	X6	X7
Mardin	1	1	0.9965	0	0.9705	1	0.9840
Sakarya	0.9562	0.9678	0.9512	0.0412	0.9274	0.6875	0.9379
Erzurum	0.9285	0.9826	0.9338	0.0398	0.9548	0.9375	0.9229
Hatay	0.9485	0.9538	0.7770	0.0552	0.8919	0.2500	0.9269
Van	0.9695	0.9886	0.9826	0.0238	0.9316	0.6563	0.9711
Urfa	0.9276	0.9593	0.8258	0.0603	0.8491	0.5625	0.9185
Tekirdag	0.9847	0.9839	0.9895	0.0069	0.9749	0.5938	1
Kocaeli	0.9034	0.9161	0.9059	0.0771	0.8741	0.7813	0.9066
Antalya	0.8900	0.8789	0.8397	0.0958	0.8612	0.4688	0.8553
Adana	0.8255	0.8950	0.8084	0.1171	0.8870	0.4375	0.8263
Eskisehir	0.9412	0.9712	1	0.0304	1	0.4688	0.9384
Bursa	0.7610	0.8454	0.7666	0.1570	0.7936	0.5938	0.7946
Denizli	0.9601	0.9737	0.9408	0.0259	0.9617	0.3125	0.9386
Antep	0.9206	0.9390	0.8188	0.0809	0.8699	0	0.8583
Ankara	0.4953	0.4680	0.3833	0.5637	0.6763	0.3750	0.3888
Istanbul	0	0	0	1	0	0.4688	0

4. Calculation of the Grey Relational Coefficients []

GRA coefficients are calculated using Equation 4 (). Grey relational coefficients are shown in Table 6.

Table 6. Grey Relational Coefficients $\gamma_{oi}(j)$.

Provinces	Variables $X_i^*(j), j=1,2,\dots,8$						
	Inputs				Outputs		
	X1	X2	X3	X4	X5	X6	X7
Mardin	0.3333	0.3333	0.3341	1	0.3400	0.3333	0.3369
Sakarya	0.3434	0.3406	0.3445	0.9238	0.3503	0.4211	0.3477
Erzurum	0.3500	0.3372	0.3487	0.9263	0.3437	0.3478	0.3514
Hatay	0.3452	0.3439	0.3915	0.9006	0.3592	0.6667	0.3504
Van	0.3403	0.3359	0.3373	0.9546	0.3493	0.4324	0.3399
Urfa	0.3502	0.3426	0.3771	0.8923	0.3706	0.4706	0.3525
Tekirdag	0.3368	0.3369	0.3357	0.9864	0.3390	0.4571	0.3333
Kocaeli	0.3563	0.3531	0.3556	0.8664	0.3639	0.3902	0.3555
Antalya	0.3597	0.3626	0.3732	0.8392	0.3673	0.5161	0.3689
Adana	0.3772	0.3584	0.3822	0.8103	0.3605	0.5333	0.3770
Eskisehir	0.3469	0.3399	0.3333	0.9427	0.3333	0.5161	0.3476
Bursa	0.3965	0.3716	0.3948	0.7610	0.3865	0.4571	0.3862
Denizli	0.3424	0.3393	0.3470	0.9508	0.3421	0.6154	0.3476
Antep	0.3520	0.3475	0.3791	0.8607	0.3650	1	0.3681
Ankara	0.5023	0.5165	0.5661	0.4700	0.4251	0.5714	0.5625
Istanbul	1	1	1	0.3333	1	0.5161	1

5. Calculation of Grey Relational assessment scores ()

The Grey relational assessment score of each province (Table 7) and the Grey relational grade for each variable (Table 5) are calculated using Equation 5

Table 7. Grey relational degrees and grades of public hospitals according to provinces.

Provinces	Γ_{0i}	Rank
Mardin	43.015%	15
Sakarya	43.878%	13
Erzurum	42.932%	16
Hatay	47.964%	4
Van	44.137%	12
Urfa	45.086%	9
Tekirdag	44.646%	11
Kocaeli	43.443%	14
Antalya	45.530%	7
Adana	45.699%	6
Eskisehir	45.451%	8
Bursa	45.055%	10
Denizli	46.922%	5
Antep	52.461%	2
Ankara	51.629%	3
Istanbul	83.564%	1

When the coefficients of the Grey relational analysis in Table 7 are reviewed, Istanbul has the highest performance among public hospitals. In other words, the public hospitals of Istanbul appear to have stronger activity structures; they are successful in the minimization of their annual hospital expenditures made from circulating capital and in the maximization of the number of surgeons and general practitioners, bed occupancy rates, operations, examinations in the emergency service and beds in clinical departments. However, this study was made at the provincial level. If hospitals in Istanbul were comparatively analysed among themselves, then efficient and inefficient hospitals would be determined with recommendations aiming at increasing the performances of inefficient hospitals.

Istanbul is among the DMUs referenced because of its strong activity structures, as determined based on the DEA results. However, Gaziantep which ranked second among the top hospital performances, was the most referenced city. Therefore, these cities were determined to be units with strong activity structures. Although Mardin and Erzurum were found to be efficient based on the DEA results, these cities ranked in the last row based on their performance statuses as assessed by the GRA. Grey relational degrees and the ranking of variables in the model are shown in Table 8.

Table 8. Grey relational degrees and ranking of variables in the model.

Variables	Γ_{0i}	Rank
Number of beds	40.204%	5
Number of surgeons	39.747%	7
Number of general practitioners	41.251%	3
Annual circulating capital	83.866%	1
Number of examinations in the emergency service	39.973%	6
Bed occupancy rates	51.531%	2
Number of operations (Group A+B+C)	40.785%	4

When the levels of the significance of factors effecting the performances of public hospitals at the provincial level were reviewed, the most effective variable was found to be the expenditures made from annual circulating capital, while bed occupancy rates and the number of general practitioners were other important variables (Table 8). Among the seven variables analysed, the number of surgeons was determined to be the least effective variable in the performances of the hospitals.

CONCLUSION

This study, which determined the efficiencies of public hospitals in the metropolitan cities of Turkey at the provincial level, aims to offer recommendations for the improvement of inefficient cities and reveal important factors to help them attain efficiency. In line with these assertions, and as a priority, the performance levels of public hospitals at the provincial level were measured using DEA to obtain efficiency scores. Based on the DEA results, modifications to be made to certain indicators in metropolitan cities that did not realize their activities efficiently and properly were demonstrated, so that these inefficient cities can offer their services correctly, just like the referenced provinces with nearly similar structural compositions.

Based on the findings gathered from the data development analyses, public hospitals in 14 metropolitan cities were not efficient at the provincial level. Therefore, these inefficient metropolitan cities should decrease their input variables and increase their effective output variables to achieve efficiency. However, this study was not conducted in collaboration with the relevant hospitals. It aims to measure and compare the total efficiency of all public hospitals at the provincial level. Within this context, the realization of an efficiency analysis by the General Secretariats of inefficient cities at the provincial level and making decisions based on the results of this analysis is thought to be a more rational approach. We think that it will be more helpful to

include equivalent data (especially of private hospitals) in comparisons and analysis, which could not be taken into consideration in this study.

GRA was employed both to rank the performances of the provincial public hospitals of 16 metropolitan cities whose efficiencies were determined using DEA and to specify the effects of variables included in the analysis on performance. The results showed that, Istanbul had the best performance, followed by Gaziantep. These provinces also had strong activity structures and were mostly referenced as assessed using DEA. However, this study was performed at the provincial level among hospitals in the metropolitan city of Istanbul; analyses among hospitals in Istanbul have been recommended to determine efficient and inefficient hospitals to increase the performances of inefficient hospitals.

Based on the ranking of the impact of variables included in the analysis of performance, annual expenditures made from circulating capital emerged as the most important variable. Two other variables effecting performance were bed occupancy rates and the number of general practitioners. As is the case in all organizations, a one-to-one correlation is present between total expenses and production outputs. As the bed occupancy rates increase, the cost per hospital bed decreases. Similarly, as bed occupancy rates increase, bed turnover intervals prolong. In other words, hospital beds remain empty for shorter periods of time and so costs of hospitalization decrease because of vacant hospital beds.

As bed turnover rate increases, especially in surgical clinics, more patients can be hospitalized for surgery, which increases the number of operations. Increase in the number of operations also increases bed occupancy rates. An inverse correlation exists between the number of general practitioners and the number of examinations in the emergency service; as the number of general practitioners increases, the number of examinations in the emergency service decreases. As the average number of patients per general practitioner increases, patients' waiting time in the emergency service increases. As a consequence, an increased number of patients leave the emergency service without being examined to consult another hospital.

Among the seven variables included in the analysis, the number of surgeons minimally effected the performances of the hospitals. Within this context, and related to the efficiency of surgeons, a study can be recommended to researchers who will investigate this issue: where, as input variables, the number of surgeons, number of beds in surgical clinics, number of operating rooms and expenditures of the personnel and, as output variables, the number of A+B+ C group operations, bed occupancy rates and operating incomes are taken into consideration.

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