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Development of Gompertz Generalized Exponential Lomax Distribution: Properties and Applications

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ABSTRACT

In Stochastic Modeling, the flexibility of probability distributions is crucial to adequately characterize vector phenomena with skewness, kurtosis, and heavy-tail (properties) behaviour. This paper presents a conceptual probability distribution referred to as Gompertz Generalized Exponential Lomax (GGEL) distribution that involves fusing the Gompertz-G family framework with the Exponential Lomax distribution. The GGEL model is the flexible one for the skewness and heavy tail of the data and thus appropriate for applications in the domains of reliability engineering, economic/financial risk assessment, survival analysis, and environmental studies. The paper verified the mathematical properties of the GGEL distribution model which are the generation of moments, quantiles, and order statistics based on the criteria of goodness-of-fit, model adequacy; Log Likelihood, AIC, and BIC values, the comparative analysis was made between the existing models of Exponential, Lomax, Exp-xponential, and Exp-Gompertz distributions. The outcomes show that the GGEL distribution considered appropriate for real life data as it clearly outperforms other distributions in term of fit and simplicity. The paper recommends the GGEL model its useful role of the combination of distributions properties in enriching the flexibility and performance of models in different fields of application.

Keywords: Distribution, Model, Probability, Properties, Stochastic

1. Introduction

In the field economic/statistical modeling, the progressive advances and formation of flexible probability distributions have greatly contributed to the accuracy of data representation, especially for real-life phenomena with the characteristics of skewness, kurtosis and heavy-tailed behavior. In this context, creating new distributions following some mathematical procedure and modifying old ones in a broad sense have been the two main routes to develop models with better flexibility and fitness.

One of these is the Gompertz-G family of distributions introduced by Morad (2017). The Author used the Gompertz distribution as a generator to increase the shape parameters and hence the flexibility of baseline distributions. Conversely, the Exponential Lomax (Lomax being responsible for the model with the heavy tail) distribution, as it had been made known by El-Bassiouny, Abdo and Shahen (2015), has undergone a very good demonstration in modeling lifetime data. This was possible because of its double character (exponential and Pareto type) and, consequently, its compliant behavior with the data. Transformer serves a dual role; on the one hand, it modifies the Lomax structure, facilitating easy analytical work on the other hand, the aspects of the Lomax structure that make it heavy-tailed are kept.

This paper proposed the Gompertz Generalized Exponential Lomax (GGEL) distribution model, created by expanding the Gompertz-G framework of Morad (2017) using the Exponential Lomax distribution. The introduced GGEL model aimed at providing good flexibility in modeling skewed, heavy-tailed, and real-life data from reliability engineering, economic/financial data, survival, and environmental data. The mathematical structure of GGEL distribution was showcased in its flexibility to fit different distributions including, Moments, Quantile Functions, and order Statistic required. In model applicability was checked with real-life secondary data of Breaking Stress of Carbon Fibers (gba) and Aircraft Windshield their comparative goodness-of-fit measures.

2. Review of Related Studies

New developments in generalized distributions have recently been putting emphasis on the idea of their uniqueness in handling data with a large number of extreme values and complex structures. Smith (2022), Johnson and Reed (2023) in their studies proposed Hybrid distributions as the Gompertz-G family are needed in the field of finance to the one of climate science. The work of Kumar and Gupta (2021) exhibits the usefulness of Generalized distributions in making the models more capable of capturing the variability and hence the maximum value of baseline models via Extreme Value Analysis.

Similarly, Alizadeh (2020) collaborated that a discrete analogue of the Gompertz-G family is indirectly paving the way for some new models, which are then being investigated by them, and the statistical properties of the latter are also checked. Yahaya (2023) proposed the Generalized Odd Gompertz-G family of distributions, looked at its statistical properties and applications. The Marshall-Olkin-Gompertz-G family of distributions was proposed in an article by Chipepa (2021); which is now positioned to offer data fitting with a greater degree of flexibility, in particular for heavy-tailed data.

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This paper proposed the development of combinations of Gompertz-G and Exponential Lomax (GGEL) distribution as a bridge of limitation of conventional models. The Cumulative Distribution Function (CDF), Probability Density Function (PDF), and Quantile Function of the GGFD, as well as its properties and the possible issues were addressed.

3 Methodology

3.3.1 The Gompertz-G Family of Distributions

The Gompertz-G family of distributions is defined by its cumulative distribution function:

$$F(x) = 1 - e^{\frac{\theta}{\gamma}\left[1-\left(1-G(x)\right)^{-\gamma}\right]} \qquad \beta \text{ and } \alpha > 0 \tag{1}$$

where: θ >0 is the scale parameter,

- γ >0 is the shape parameter,
- G(x) is the CDF of the baseline distribution.

The corresponding PDF is given by:

$$f(x) = \theta g(x)\left(1 - G(x)\right)^{-\gamma-1} e^{\frac{\theta}{\gamma}\left[1-\left(1-G(x)\right)^{-\gamma}\right]} \tag{2}$$

where g(x) is the PDF of the baseline distribution.

3.3.2 The Exponential Lomax Distribution

The Exponential Lomax distribution is widely used in modeling extreme values. Its CDF, PDF, and quantile function are given as:

CDF:

$$F(x) = 1 - e^{\lambda\left(\frac{x+\beta}{\beta}\right)^{\alpha}} \tag{3}$$

where λ, β > 0 is the scale parameter and $\theta > 0$ is the shape parameter.

PDF:

$$g_{EL}(x) = \frac{\alpha\lambda}{\beta}\left(\frac{x+\beta}{\beta}\right)^{\alpha-1} e^{\lambda\left(\frac{x+\beta}{\beta}\right)^{\alpha}} \qquad x > 0.\ \alpha, \lambda\ and\ \beta > 0 \tag{4}$$

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3.3.3 Quantile Function:

$$Q_{EL}(x) = \beta\left[\left(\frac{-\ln(1-p)}{\lambda}\right)^{\frac{1}{\alpha}} - 1\right] \tag{5}$$

$$Median(x) = \beta\left[\left(\frac{-\ln(0.5)}{\lambda}\right)^{\frac{1}{\alpha}} - 1\right] \tag{6}$$

3.3.4 The Gompertz Generalized Exponential Lomax Distribution

Using the Gompertz-G framework, the GGFD is derived with the Exponential Lomax distribution as the baseline. Substituting G(x) and g(x) from the Exponential Lomax distribution into the Gompertz-G formula, we obtain:

CDF:

$$F(x) = 1 - e^{\frac{\theta}{\gamma}\left[1-e^{\lambda\gamma\left(\frac{x+\beta}{\beta}\right)^{\alpha}}\right]} \tag{7}$$

PDF:

$$f(x) = \frac{\alpha\theta\lambda}{\beta}\left(\frac{x+\beta}{\beta}\right)^{\alpha-1} e^{\lambda\gamma\left(\frac{x+\beta}{\beta}\right)^{\alpha}} e^{\frac{\theta}{\gamma}\left[1-e^{\lambda\gamma\left(\frac{x+\beta}{\beta}\right)^{\alpha}}\right]} \tag{8}$$

From equation 8, Using Taylor series expansion

$$e^{\frac{\theta}{\gamma}\left[1-e^{\lambda\gamma\left(\frac{x+\beta}{\beta}\right)^{\alpha}}\right]} = \sum_{i=o}^{\infty}\frac{\left(\frac{\theta}{\gamma}\right)^{i}}{i}\left(1-e^{\lambda\gamma\left(\frac{x+\beta}{\beta}\right)^{\alpha}}\right)^{i} \tag{9}$$

$$\sum_{i=o}^{\infty}\frac{\left(\frac{\theta}{\gamma}\right)^{i}}{i}\left(1-e^{\lambda\gamma\left(\frac{x+\beta}{\beta}\right)^{\alpha}}\right)^{i} = \sum_{i=o}^{\infty}\frac{\left(\frac{\theta}{\gamma}\right)^{i}}{i}\sum_{j=0}^{\infty}(-1)^{j}\binom{i}{j}\left(e^{\lambda\gamma\left(\frac{x+\beta}{\beta}\right)^{\alpha}}\right)^{j} \tag{10}$$

Substitute equation 9 and 10 in equation 8

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$$f(x) = \frac{\alpha\theta\lambda}{\beta}\left(\frac{x+\beta}{\beta}\right)^{\alpha-1} e^{\lambda\gamma\left(\frac{x+\beta}{\beta}\right)^{\alpha}} \sum_{i=o}^{\infty}\frac{\left(\frac{\theta}{\gamma}\right)^{i}}{i}\sum_{j=0}^{\infty}(-1)^{j}\binom{i}{j}\left(e^{\lambda\gamma j\left(\frac{x+\beta}{\beta}\right)^{\alpha}}\right)$$

$$f(x) = \frac{\alpha\theta\lambda}{\beta}\left(\frac{x+\beta}{\beta}\right)^{\alpha-1} \sum_{i=o}^{\infty}\sum_{j=1}^{\infty}\frac{\left(\frac{\theta}{\gamma}\right)^{i}}{i}(-1)^{j}\binom{i}{j} e^{(\lambda\gamma(j+1))\left(\frac{x+\beta}{\beta}\right)^{\alpha}}$$

$where\, m = \lambda\gamma(j+1)$

and

$$w_i = \sum_{i=o}^{\infty}\sum_{j=1}^{\infty}\frac{\left(\frac{\theta}{\gamma}\right)^{i}}{i}(-1)^{j}\binom{i}{j}$$

$$f(x) = \frac{\alpha\theta\lambda}{\beta}\left(\frac{x+\beta}{\beta}\right)^{\alpha-1} w_i e^{m\left(\frac{x+\beta}{\beta}\right)^{\alpha}} \quad (11)$$

3.3.5 Distribution Curves

Fig. 1: CDF of Gompertz Generalized Exponential Lomax Distribution

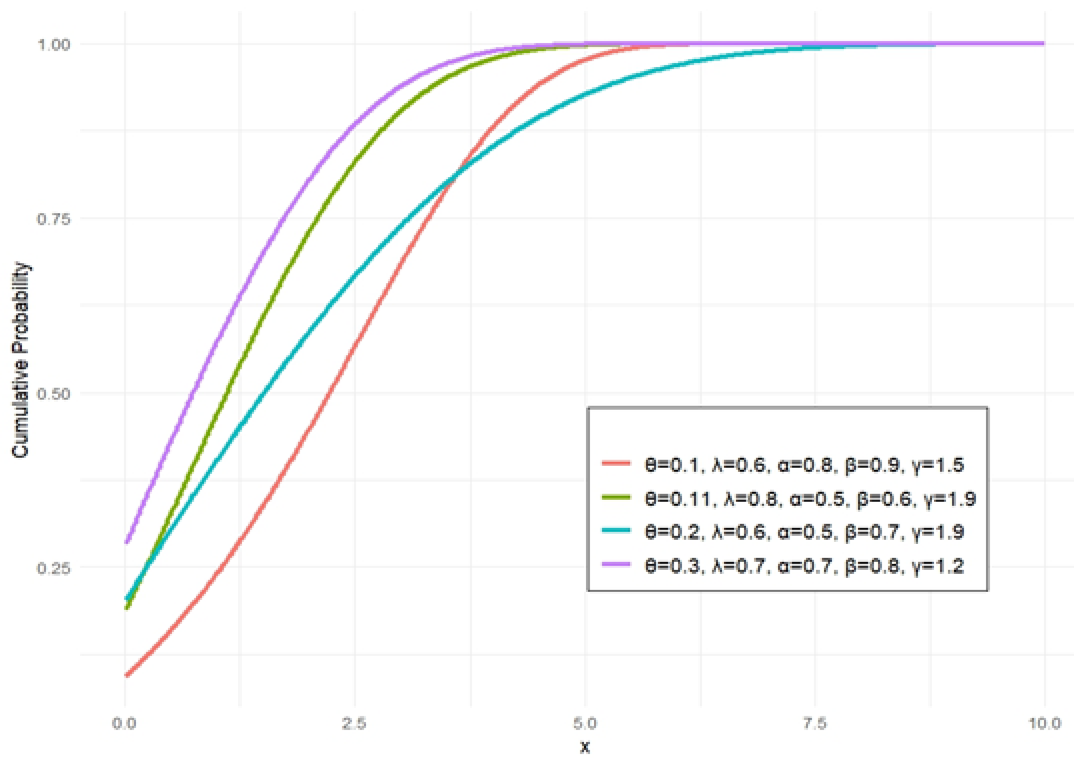

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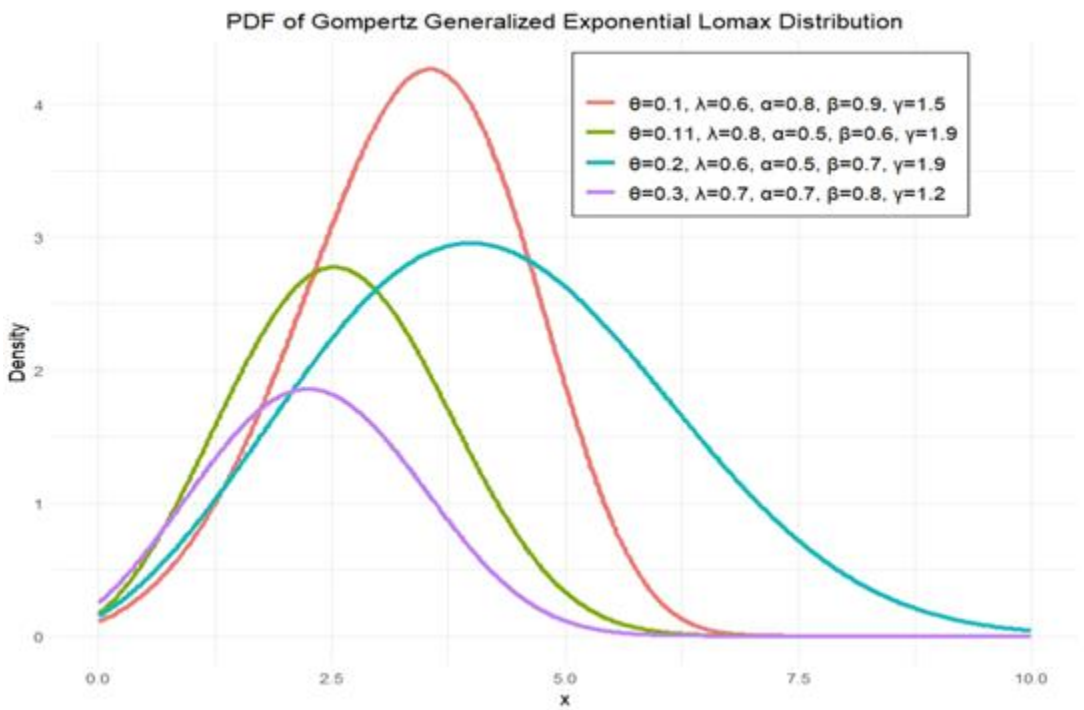
Fig. 2: PDF of Gompertz Generalized Exponential Lomax Distribution



3.3.6 Parameter Estimation

The parameters of the EGG-F distribution are estimated using the maximum likelihood estimation (MLE) method. Let $x_1,x_2,\ldots,x_n,$be a random sample from the GG-EL distribution. The likelihood function L(θ,λ,α,β,γ) for a sample $x_1,x_2,\ldots,x_n$ is the product of the PDFs of each individual observation:

$$L(x;\alpha,\theta,\beta,\lambda,\gamma)$$
$$=\prod_{i=1}^{n} f(x;\alpha,\theta,\beta,\lambda) \quad (11)$$

When substitute pdf into MLE

$$f(x)=\frac{\alpha\theta\lambda}{\beta}\left(\frac{x+\beta}{\beta}\right)^{\alpha-1}\sum_{i=o}^{\infty}\sum_{j=1}^{\infty}\frac{\left(\frac{\theta}{\gamma}\right)^{i}}{i}(-1)^{j}\binom{i}{j}e^{(\lambda\gamma(j+1))\left(\frac{x+\beta}{\beta}\right)^{\alpha}}$$

$$L(x;\alpha,\theta,\beta,\lambda,\gamma) = \prod_{i=1}^{n}\left(\frac{\alpha\theta\lambda}{\beta}\left(\frac{x+\beta}{\beta}\right)^{\alpha-1} e^{\lambda\gamma\left(\frac{x+\beta}{\beta}\right)^{\alpha}} e^{\frac{\theta}{\gamma}\left[1-e^{\lambda\gamma\left(\frac{x+\beta}{\beta}\right)^{\alpha}}\right]}\right) \quad (12)$$

Now, take the natural logarithm to get the log-likelihood function:

$$LogL(\theta,\lambda,\alpha,\beta,\gamma) = \sum_{i=1}^{n}\left(log\theta + log\lambda + log\alpha - log\beta + (\alpha-1)log\left(\frac{x_i+\beta}{\beta}\right) + \lambda\gamma\left(\frac{x_i+\beta}{\beta}\right)^{\alpha} + \frac{\theta}{\gamma}\left(1-e^{-\lambda\left(\frac{x_i+\beta}{\beta}\right)^{\alpha}}\right)\right) \quad (13$$

Distribute summation;

$$L(\theta,\lambda,\alpha,\beta,\gamma) = \left(nlog\theta + nlog\lambda + nlog\alpha - nlog\beta + (\alpha-1)\sum_{i=1}^{n} log\left(\frac{x_i+\beta}{\beta}\right) - \lambda\gamma\sum_{i=1}^{n}\left(\frac{x_i+\beta}{\beta}\right)^{\alpha} + \frac{\theta}{\gamma}\sum_{i=1}^{n}\left(1-e^{\gamma\lambda\left(\frac{x_i+\beta}{\beta}\right)^{\alpha}}\right)\right) \quad (14$$

Partial Derivative of eqn 14 wrt associated parameters;

$$\frac{\partial\ell}{\partial\theta} = \frac{n}{\theta} - \frac{1}{\gamma}\sum_{i=1}^{n}\left(1-e^{-\lambda\left(\frac{x_i+\beta}{\beta}\right)^{\alpha}}\right)^{-\gamma}$$

$$\frac{\partial\ell}{\partial\lambda} = \frac{n}{\lambda} + \gamma\sum_{i=1}^{n}\left(\frac{x_i+\beta}{\beta}\right)^{\alpha} - \theta\sum_{i=1}^{n}\left(\frac{x_i+\beta}{\beta}\right)^{\alpha} e^{\gamma\lambda\left(\frac{x_i+\beta}{\beta}\right)^{\alpha}}$$

$$\frac{\partial\ell}{\partial\alpha} = \frac{n}{\alpha} + \sum_{i=1}^{n} log\left(\frac{x_i+\beta}{\beta}\right) - \lambda\gamma\sum_{i=1}^{n}\left(\frac{x_i+\beta}{\beta}\right)^{\alpha} log\left(\frac{x_i+\beta}{\beta}\right) - \theta\lambda\sum_{i=1}^{n}\left(\frac{x_i+\beta}{\beta}\right)^{\alpha} log\left(\frac{x_i+\beta}{\beta}\right)\left(e^{\gamma\lambda\left(\frac{x_i+\beta}{\beta}\right)^{\alpha}}\right)$$

$$\frac{\partial\ell}{\partial\beta} = -\frac{n}{\beta} + (\alpha-1)\sum_{i=1}^{n}\left(\frac{1}{x_i+\beta} - \frac{1}{\beta}\right) - \alpha\gamma\lambda\sum_{i=1}^{n} x_i\left(\frac{x_i+\beta}{\beta}\right)^{\alpha-1}\frac{1}{\beta^2} + \alpha\sum_{i=1}^{n} x_i\left(\frac{x_i+\beta}{\beta}\right)^{\alpha-1}\frac{1}{\beta^2}\left(e^{\gamma\lambda\left(\frac{x_i+\beta}{\beta}\right)^{\alpha}}\right)$$

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$$\frac{\partial \ell}{\partial \gamma} = \lambda \sum_{i=1}^{n} \left(\frac{x_i + \beta}{\beta}\right)^{\alpha} - \frac{\theta}{\gamma^2} \sum_{i=1}^{n} \left(1 - e^{\lambda\gamma\left(\frac{x_i+\beta}{\beta}\right)^{\alpha}}\right) - \frac{\lambda\theta}{\gamma} \sum_{i=1}^{n} \left(\frac{x_i + \beta}{\beta}\right)^{\alpha} \left(e^{\lambda\gamma\left(\frac{x_i+\beta}{\beta}\right)^{\alpha}}\right)$$

4. Applications Results

The GG-EL distribution is applied to two real-world datasets to demonstrate its modeling capabilities. The performance GG-EL model was compared with six other competing models; Statistical Fit Criteria adopted were Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC) and Log Likelihood (LL) and Kolmogorov-Smirnov (KS), Anderson-Darling (AD), and Cramér-von Mises (CM) statistics were used to test models' Goodness-of-Fit Criteria.

4.1. Data:

Uncensored data set consisting of 100 observations on Breaking Stress of Carbon Fibers (in Gba) and 85 Aircraft Windshield were used as training data for the GG-LE modeling.

4.2. Results

Table 1: Summary Statistics

	Mean	SD	Median	Min	Max	Skew	Kurtosis	Range
Breaking stress of carbon fibers	2.62	1.01	2.70	0.39	5.56	0.36	0.04	5.17
Breaking stress of Aircraft Windshield	2.257	2.22	0.840	0.04	4.66	4.62	0.33	2.77

This table provides **key descriptive statistic the mean, standard deviation, median, minimum, maximum, skewness, kurtosis, and range**.

Table 2: Model Selection Criteria Statistic

	LogLikelihood		AIC		BIC	
Distribution	**Carbon Fibers**	**Aircraft Wind Shield**	**Carbon Fibers**	**Aircraft Wind Shield**	**Carbon Fibers**	**Aircraft Wind Shield**
Exponential	295.68	251.02	593.36	504.04	595.96	506.49
Exp-Exponential	787.83	662.31	1579.67	1328.63	1584.88	1333.51
Lomax	258.44	213.90	520.88	431.79	526.09	436.68
Exp-Exp-Lomax	729.01	618.30	1466.02	1244.60	1476.44	1254.37
Gompertz	311.32	238.61	626.66	481.23	631.87	486.11

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Exp-Gompertz	944.25**	764.42**	1894.52**	1534.84**	1902.33**	1542.17**
GGEL	217.04*	183.04*	444.08*	376.09*	457.10*	388.30*

* GGEL has the smallest values. Hence considered best model

** Exp-Gompertz has the highest values. issue of Over-fitting

Table 3: Goodness-of-fit Criterion

	KS_		**AD_**		**CM_**	
Distribution	**Carbon Fibers**	**Aircraft Wind Shield**	**Carbon Fibers**	**Aircraft Wind Shield**	**Carbon Fibers**	**Aircraft Wind Shield**
Exponential	0.95	0.93	722.71	621.58	32.62	26.17
Exp_Exponential	0.92	0.91	610.43	528.94	31.57	24.98
Lomax	0.99**	0.99**	797.72**	694.39**	33.32**	28.32**
Exp_Exp_Lomax	0.94	0.96	194.56	189.18	28.08	24.96
Gompertz	0.95	0.93	722.71	621.58	32.62	26.17
Exp_Gompertz	0.92	0.91	610.44	528.94	31.57	24.90
GGEL	0.91*	0.80*	149.54*	141.02*	25.09*	22.38*

* GGEL has the smallest values. Hence considered best model

** Lomax has the highest values

4.3 Histogram Plots

Fig. 3: Histogram with Theoretical Density of Breaking Stress of Carbon Fibers

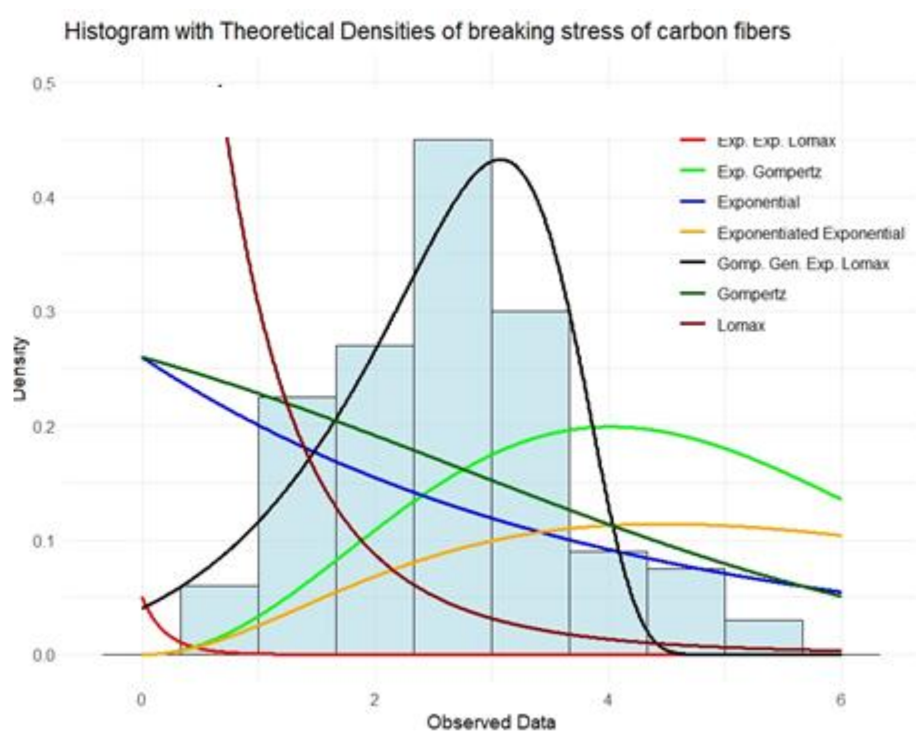

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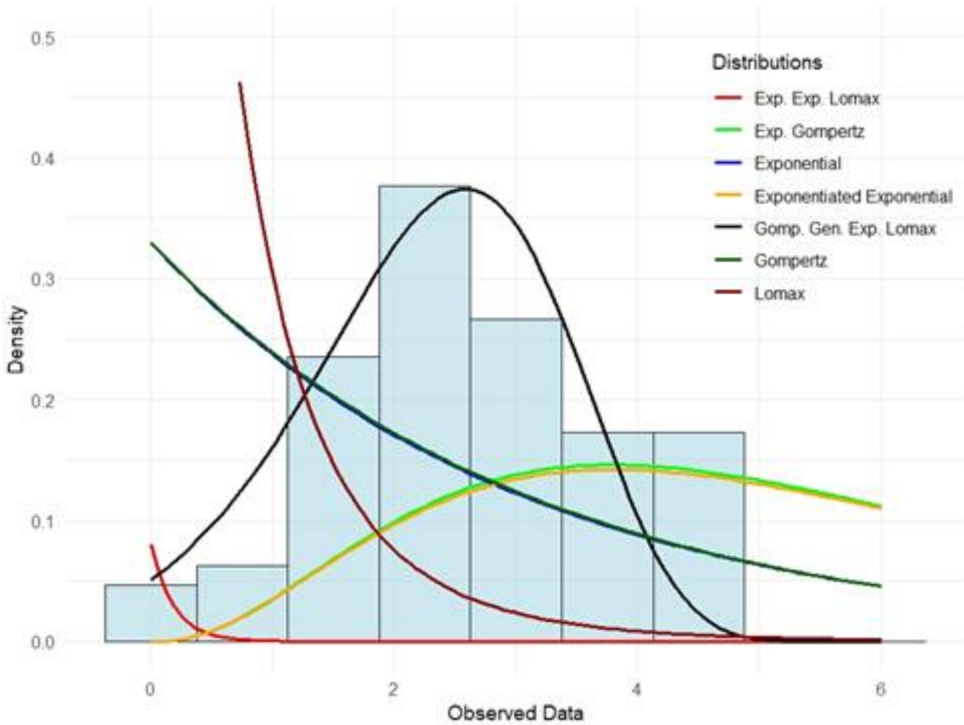
Fig. 4: Histogram with Theoretical Densities of Aircraft Windshield



5. Discussions

Table 1 shows the descriptive statistic of the training real-life data used from two applications; Breaking Stress of Carbon Fibers (in Gba) and Aircraft Windshield. Table 2 shows the combined results of Log Likelihood (in absolute value), Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC) values of probability distributions of fitted models. These values are commonly used as means of assessing and compare model selection; the model with the smallest value is considered best.

The Lomax distribution with a better Log Likelihood, AIC, and BIC value than the Exponential and Exp-Exponential distributions indicating, a more plausible fit for the data but still an improvement is needed over other models. The AIC and BIC values for this distribution are larger than those for the Lomax model, suggesting that the addition of the exponential component adds complexity without meaningful improvement in fit. The AIC and BIC values of the Gompertz distribution are quite high, and indicate that this may not be the best fitting model for the data. It does not seem to perform better than the Lomax or GGEL distributions. such that the purpose of having an exponential and Gompertz component does not contribute significantly

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to the fit as indicated from the AIC and BIC values, where the Exp-Gompertz models obtain the highest AIC and BIC values in this comparison. This model is probably overfit.

The Plots

Fig. 3 plot revealed that the Gompertz Generalized Exponential Lomax (GGEL) distribution, provide the best fit as the curve depicted normal curve, closely aligning with the observed data in both the peak and tail areas. Other distributions, such as the Lomax and Exponential Gompertz, either overestimate or underestimate the data in key regions as well as in Fig. 4.

6. Conclusion

The analysis of the six probability distributions considered applied to fitting the breaking stress of carbon fibers and the failure times of aircraft windshields reveals that the GGEL (Gompertz Generalized Exponential Lomax) distribution consistently outperforms other models. It provides the best fit across all the evaluated criteria, including Log Likelihood, AIC, BIC, KS Statistic, AD Statistic, and CM Statistic. The GGEL distribution not only aligns with the data in both the central peak and tail but also strikes a balance between model complexity and fit, making it the optimal choice for both datasets.

7. Recommendations

The paper recommends as follows;

1. GGEL for Modeling be considered in modeling the breaking stress of carbon fibers and failure times of aircraft windshields. Its ability to capture both the peak and tail of the observed data makes it ideal for these types of analyses.

2. Lomax distribution for simplicity can be considered as a secondary option, especially if a simpler model is required. Although it is not as good as GGEL, it still provides a reasonable fit and may be more computationally efficient in certain cases.

3. Use of Complex Models; Exp-Gompertz, Exp-Exponential, and Exp-Exp-Lomax may be avoided, as they tend to overfit the data without offering a meaningful improvement in the model's predictive capabilities.

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